

Variable Dependencies & Q-resolution

Friedrich Slivovsky & Stefan Szeider

PCNF $\exists x_1 \exists x_2 \exists x_3 \forall y_1 \forall y_2 \exists z_1 \exists z_2 \exists z_3 F$

plain QDPLL

$E_{X_1} E_{X_2} E_{X_3} \forall y_1 \forall y_2 E_{Z_1} E_{Z_2} E_{Z_3}$

plain QDPLL

$\exists X_1 \quad \exists X_3 \forall y_1 \forall y_2 \exists Z_1 \exists Z_2 \exists Z_3$

plain QDPLL

$\exists x_1 \quad \forall y_1 \forall y_2 \exists z_1 \exists z_2 \exists z_3$

plain QDPLL

$\forall y_1 \forall y_2 \exists z_1 \exists z_2 \exists z_3$

plain QDPLL

$\forall y_2 \exists z_1 \exists z_2 \exists z_3$

plain QDPLL

$z_1 z_2 z_3$

plain QDPLL

$\exists Z_1$ $\exists Z_3$

plain QDPLL

$\exists Z_3$

plain QDPLL

DepQBF

$\exists x_1 \exists x_2 \exists x_3 \forall y_1 \forall y_2 \exists z_1 \exists z_2 \exists z_3 F$

DepQBF

$\exists X_1 \exists X_2 \exists X_3 \forall y_1 \forall y_2 \exists Z_1 \exists Z_2 \exists Z_3 F$

Dependency Scheme

DepQBF

$\exists X_1 \exists X_2 \exists X_3 \forall y_1 \forall y_2 \exists Z_1 \exists Z_2 \exists Z_3 F$



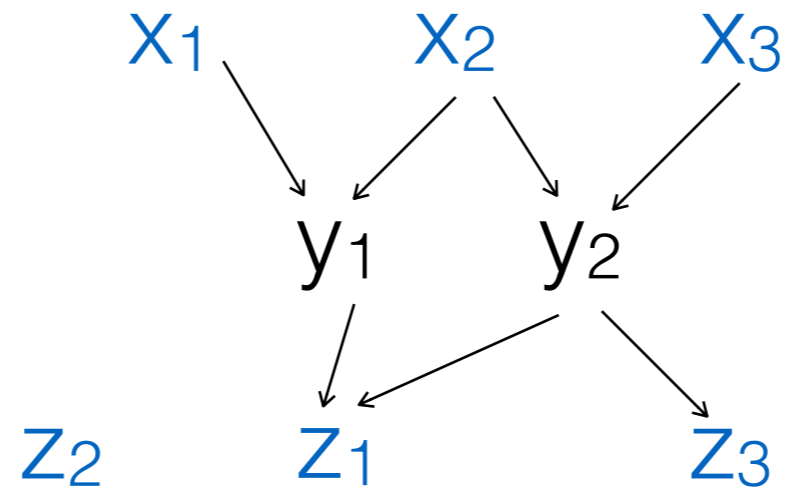
Dependency Scheme

DepQBF

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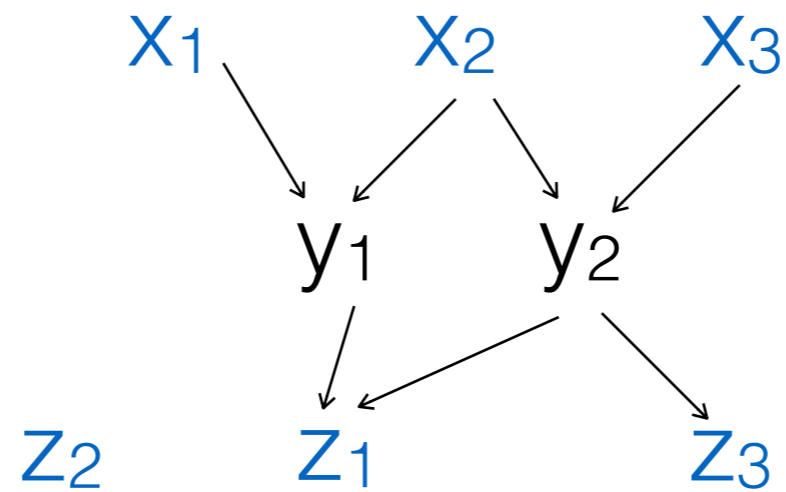


Dependency Scheme



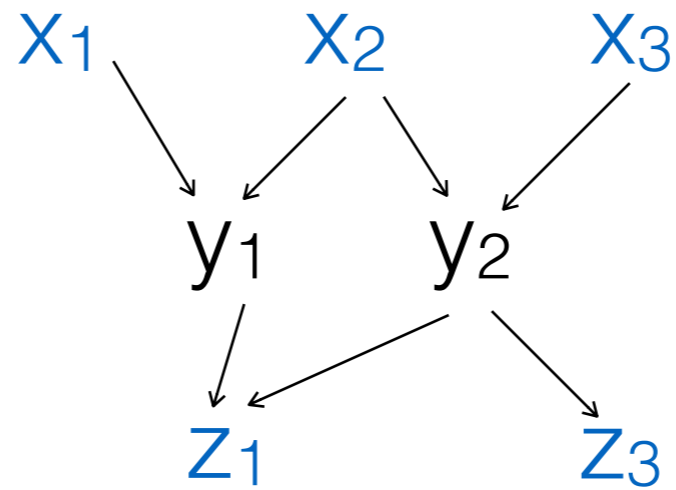
DepQBF

$\exists X_1 \exists X_2 \exists X_3 \forall y_1 \forall y_2 \exists Z_1 \exists Z_2 \exists Z_3$



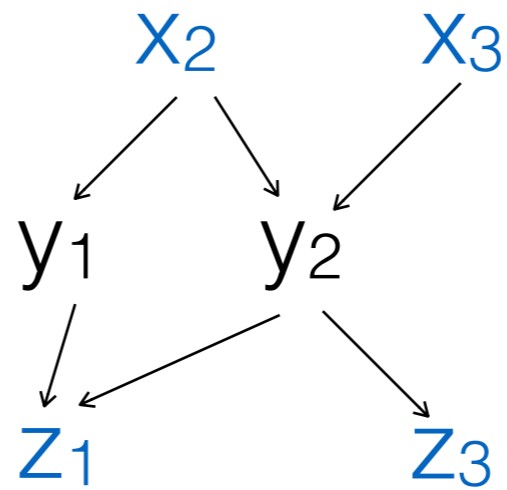
DepQBF

$\exists X_1 \exists X_2 \exists X_3 \forall y_1 \forall y_2 \exists Z_1 \exists Z_3$



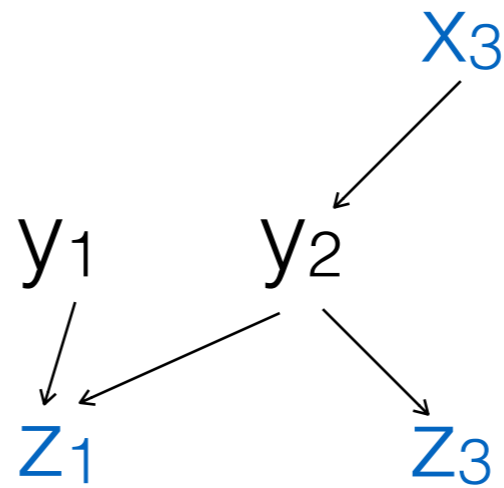
DepQBF

$\exists X_2 \exists X_3 \forall y_1 \forall y_2 \exists Z_1 \exists Z_3$



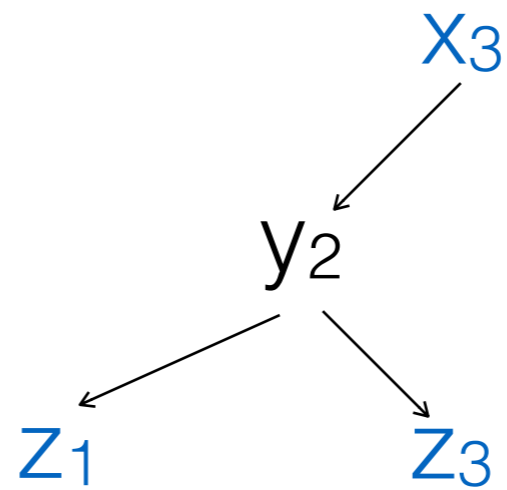
DepQBF

$\exists x_3 \forall y_1 \forall y_2 \exists z_1 \quad \exists z_3$



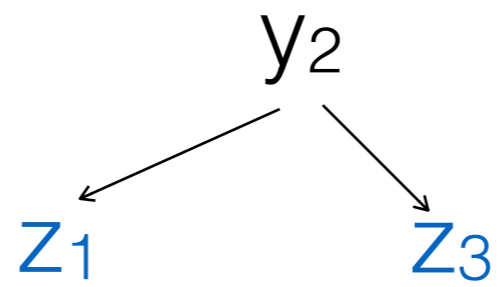
DepQBF

$\exists X_3 \quad \forall y_2 \exists Z_1 \quad \exists Z_3$



DepQBF

$\forall y_2 \exists z_1 \exists z_3$



DepQBF

$\exists Z_1 \quad \exists Z_3$

Z_1

Z_3

DepQBF

$\exists Z_3$

Z_3

DepQBF

DepQBF

more freedom in decision making

DepQBF

more freedom in decision making

shorter learned clauses

DepQBF

more freedom in decision making

shorter learned clauses

more unit clauses

DepQBF
prefix order

formula

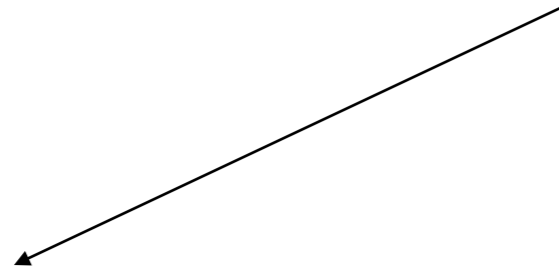


DepQBF
prefix order

formula



DepQBF
prefix order



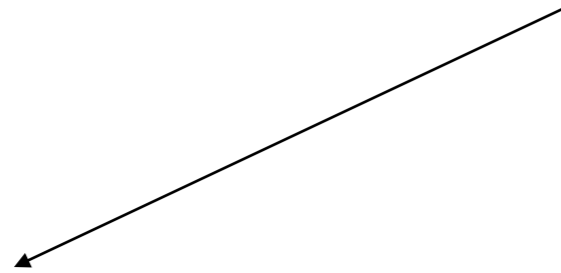
true

Q-Term resolution proof

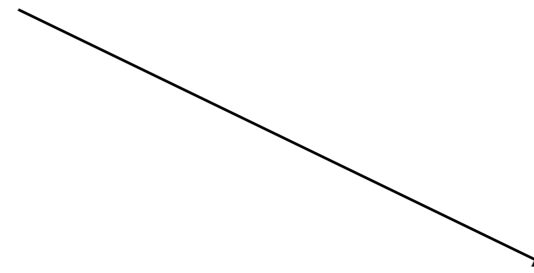
formula



DepQBF
prefix order



true



false

Q-Term resolution proof

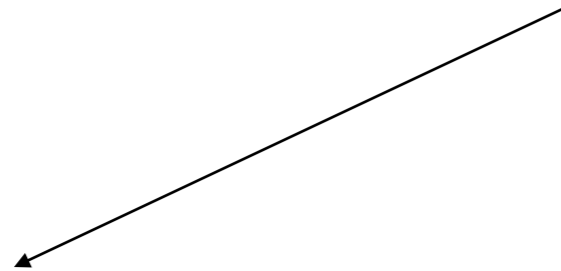
Q-resolution refutation

formula

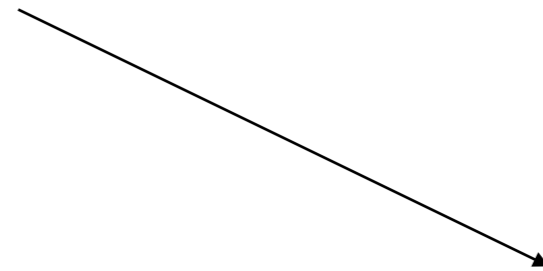


DepQBF

dependency scheme **D**



true



false

Q(**D**)-Term resolution proof

Q(**D**)-resolution refutation

Cumulative Dependency Schemes

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allow sound [reordering](#).

Cumulative Dependency Schemes

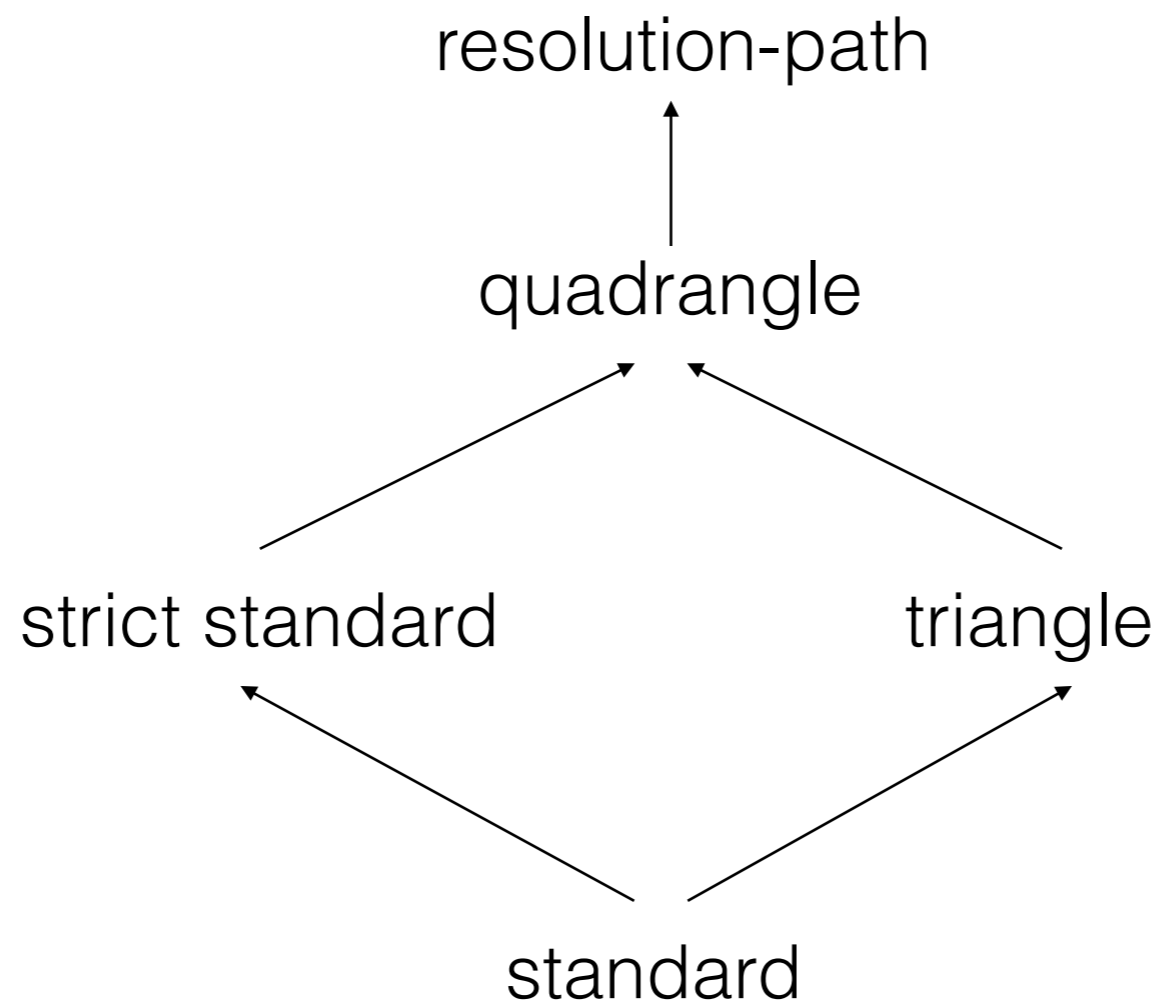
allow sound [reordering](#).

But Q(D)-resolution can be **unsound**.

Cumulative Dependency Schemes

allow sound **reordering**.

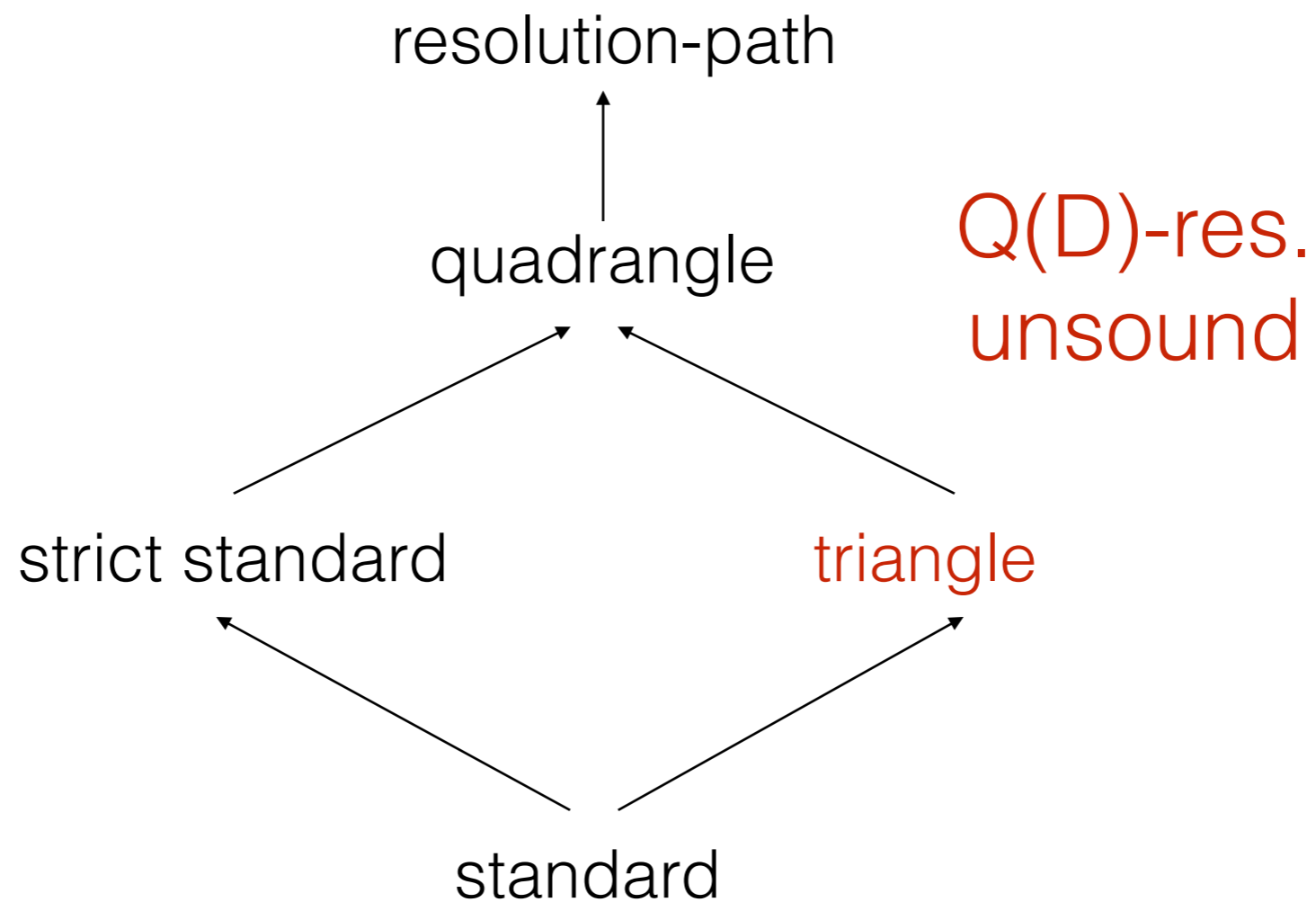
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Cumulative Dependency Schemes

allow sound **reordering**.

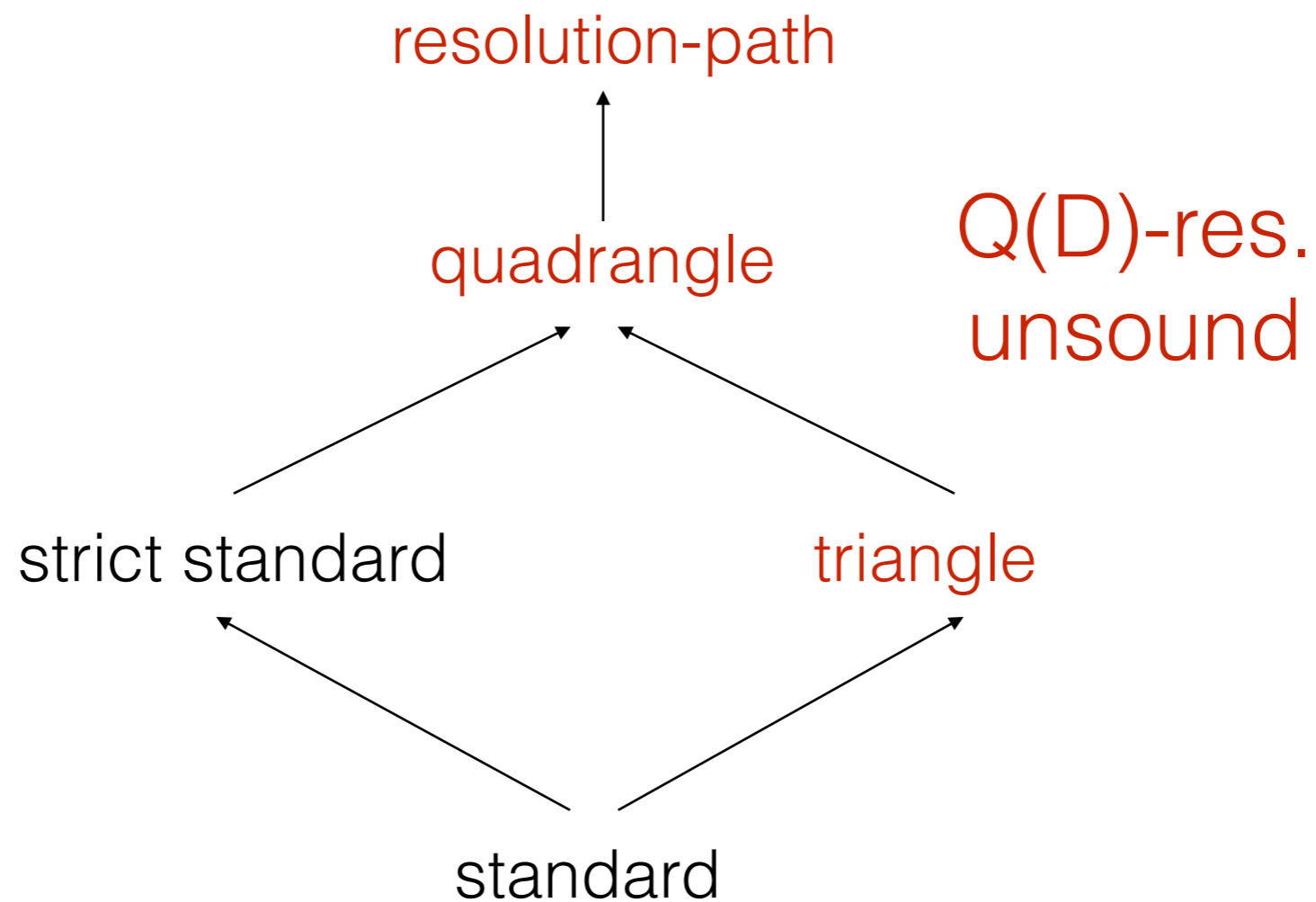
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Cumulative Dependency Schemes

allow sound **reordering**.

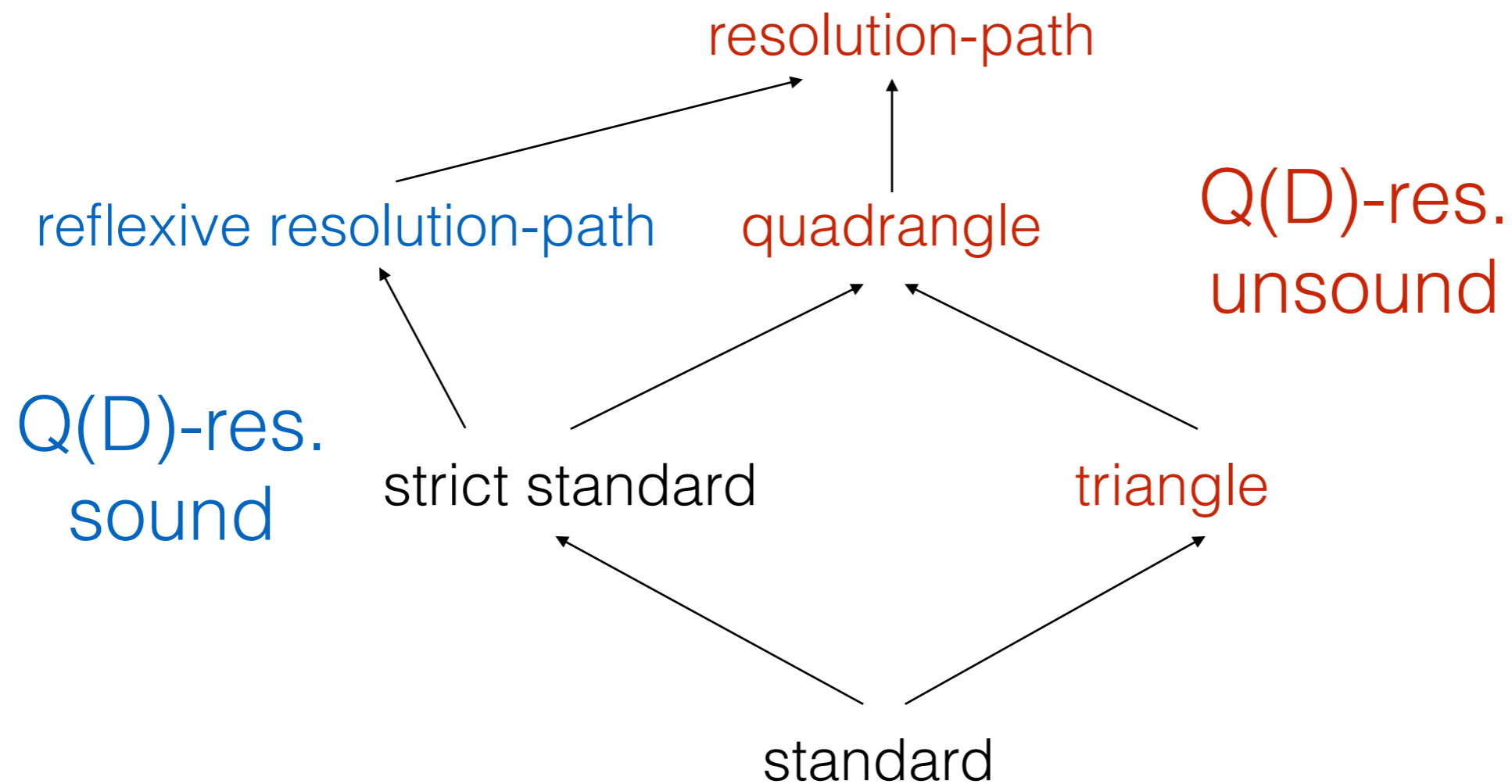
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Cumulative Dependency Schemes

allow sound **reordering**.

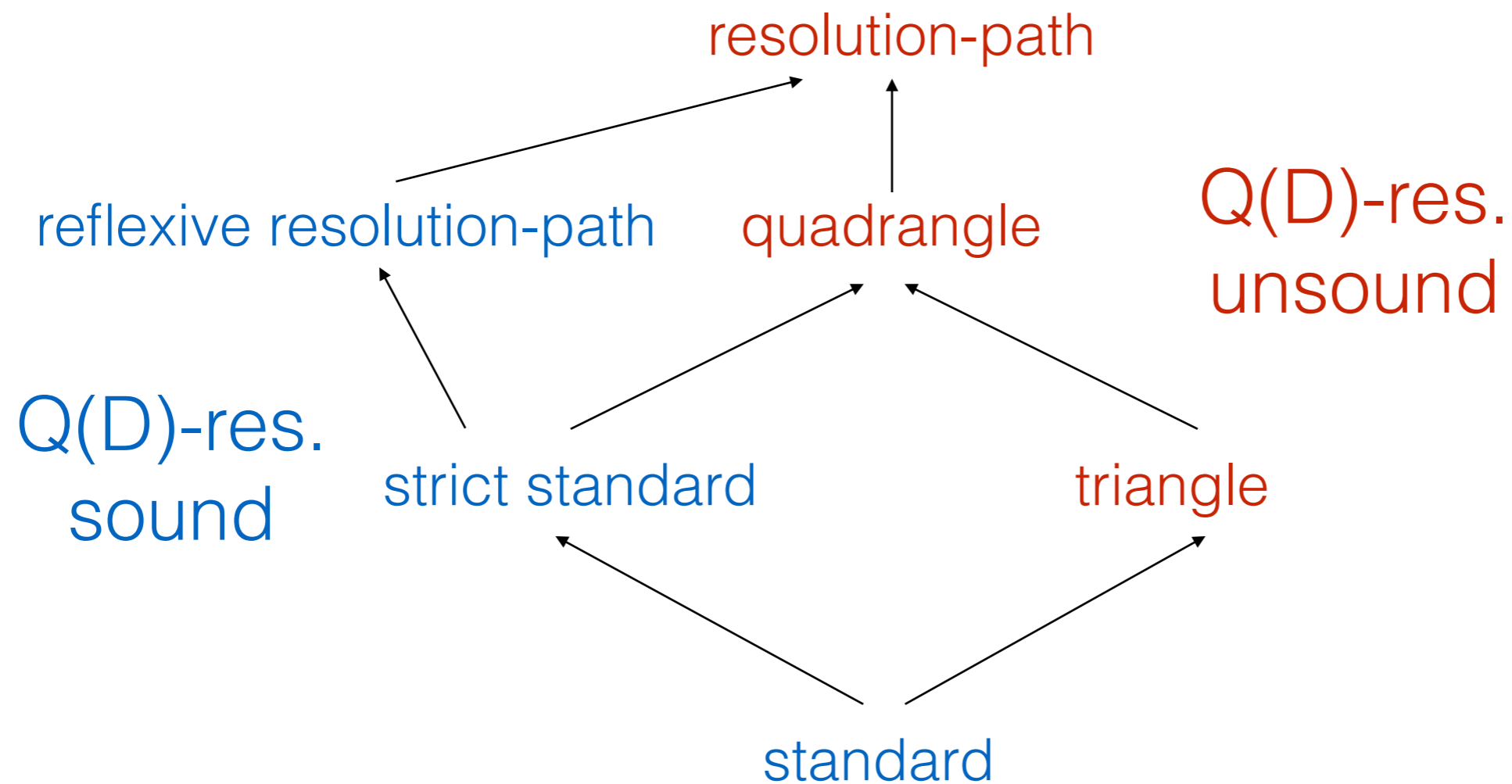
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Cumulative Dependency Schemes

allow sound **reordering**.

But Q(D)-resolution can be **unsound**.



Our Result(s)

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Q(**Drrs**)-resolution is sound.

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(reflexive) resolution-path dependency scheme



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Corollary

Q(**Dstd**)-resolution is sound.

Our Result(s)

Q(**Drrs**)-resolution is sound.

(reflexive) resolution-path dependency scheme



Corollary

Q(**Dstd**)-resolution is sound.

standard dependency scheme



Q-resolution

Q-resolution

$$\exists e \frac{C \vee e \quad \neg e \vee C'}{C \vee C'} \quad \text{resolution}$$

Q-resolution

$$\exists e \frac{C \vee e \quad \neg e \vee C'}{C \vee C'}$$

resolution

$$\forall u \frac{C \vee u}{C}$$

universal reduction

Q-resolution

$$\exists e \frac{C \vee e \quad \neg e \vee C'}{C \vee C'}$$

resolution

$$\forall u \frac{C \vee u}{C}$$

universal reduction

$$Q_1 X_1 \dots \forall u \dots Q_n X_n$$

Q-resolution

$$\exists e \frac{C \vee e \quad \neg e \vee C'}{C \vee C'}$$

resolution

$$\forall u \frac{C \vee u}{C}$$

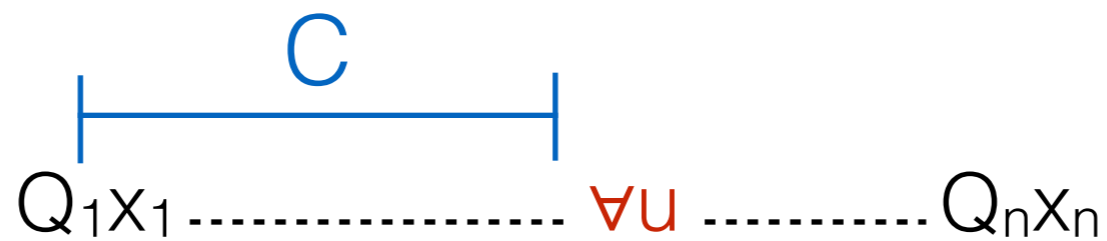
universal reduction



Q-resolution

$$\exists e \frac{C \vee e \quad \neg e \vee C'}{C \vee C'} \quad \text{resolution}$$

$$\forall u \frac{C \vee u}{C} \quad \text{universal reduction}$$



sound and complete for PCNF formulas

$\forall z \exists x \forall u \exists y \exists e$

$z \forall x \forall y$

$y \forall e$

$u \forall \neg e$

$\neg x \forall \neg u$

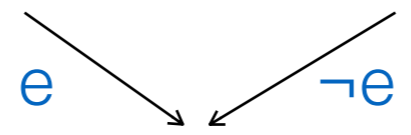
$\forall z \exists x \forall u \exists y \exists e$

$z \vee x \vee \neg y$

$y \vee e$

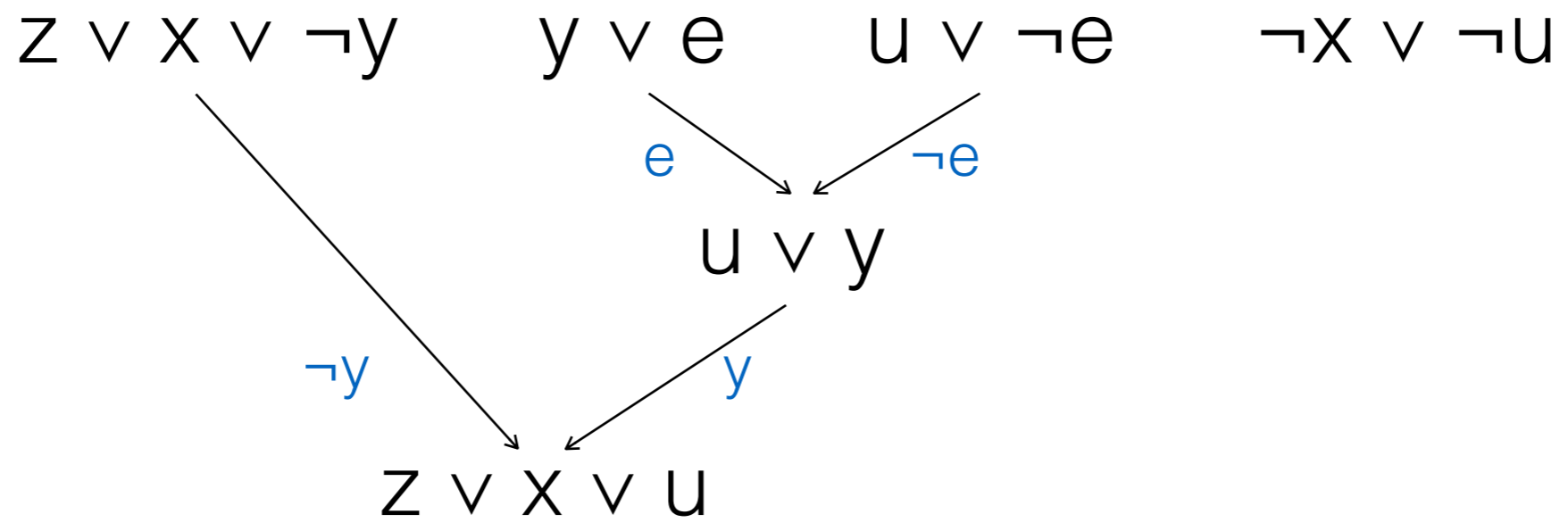
$u \vee \neg e$

$\neg x \vee \neg u$

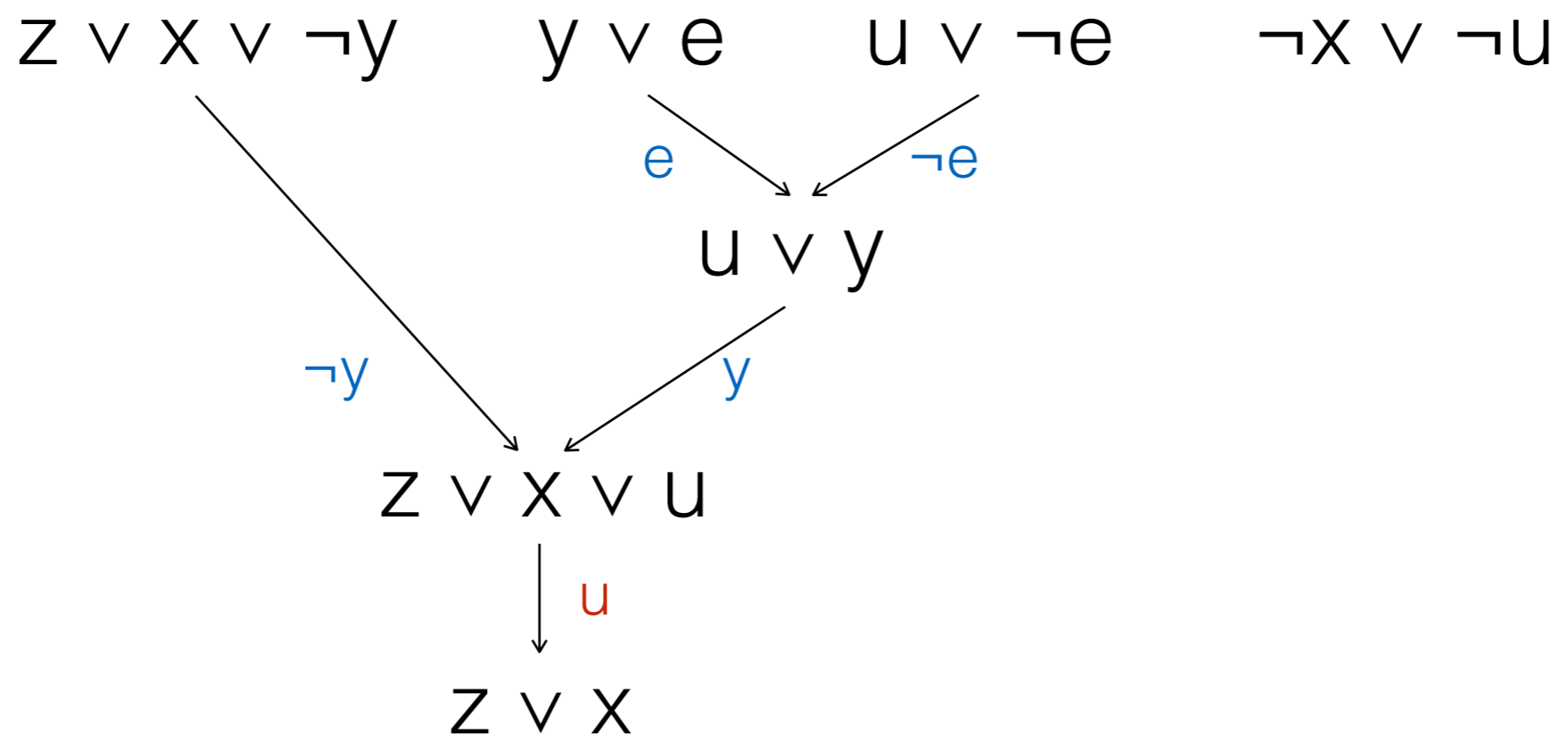


$u \vee y$

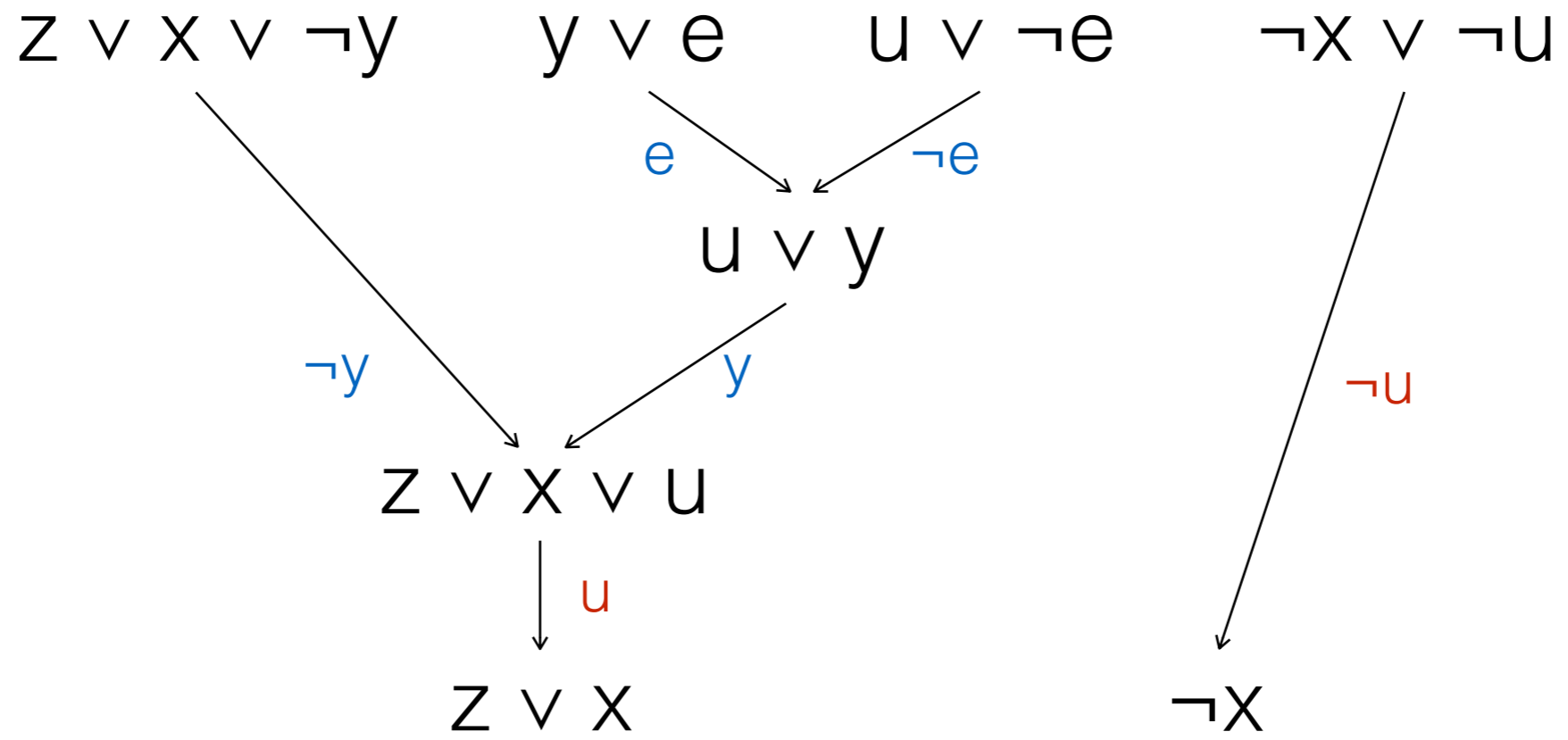
$\forall z \exists x \forall u \exists y \exists e$



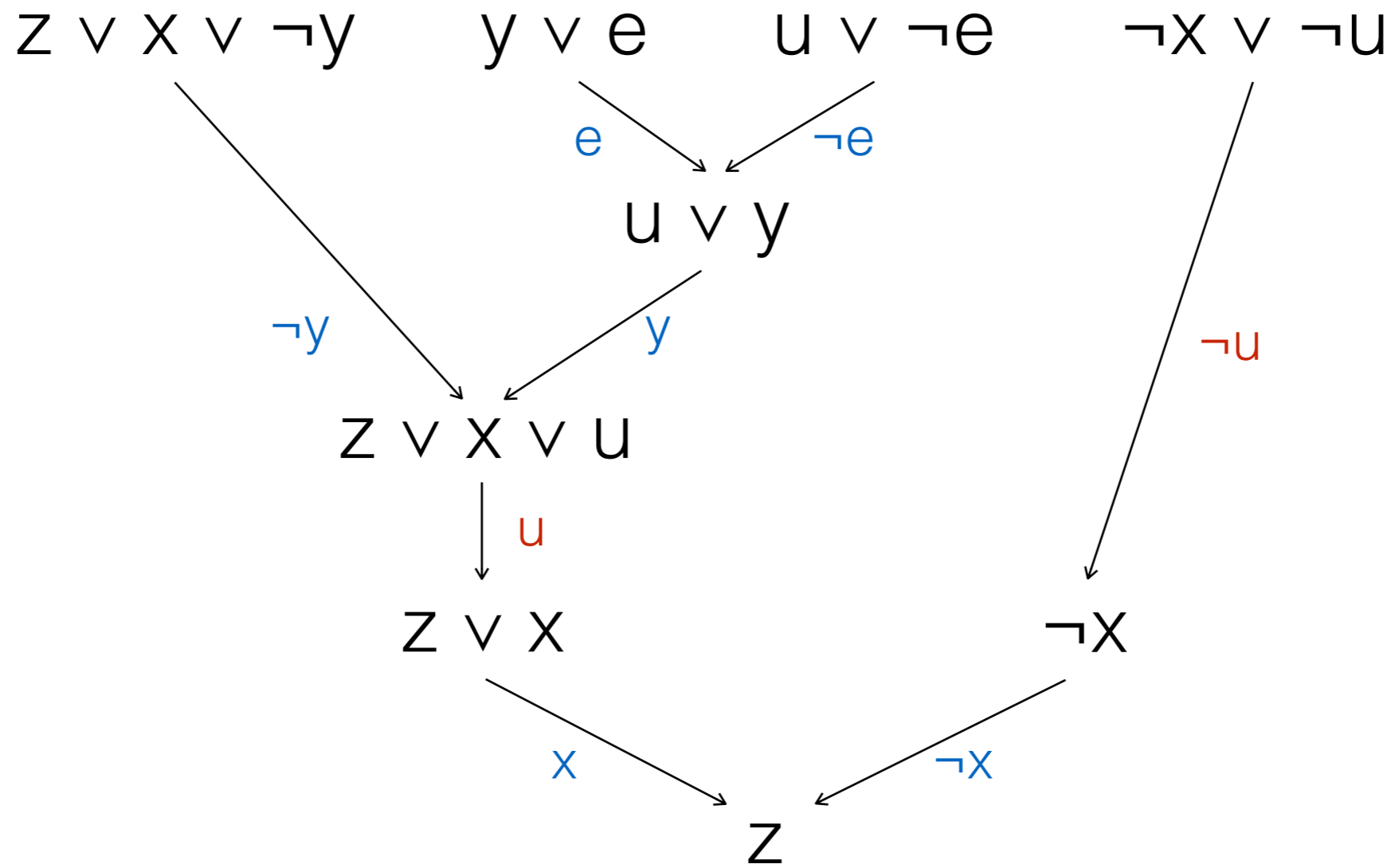
$\exists e \exists u \exists y \exists z$



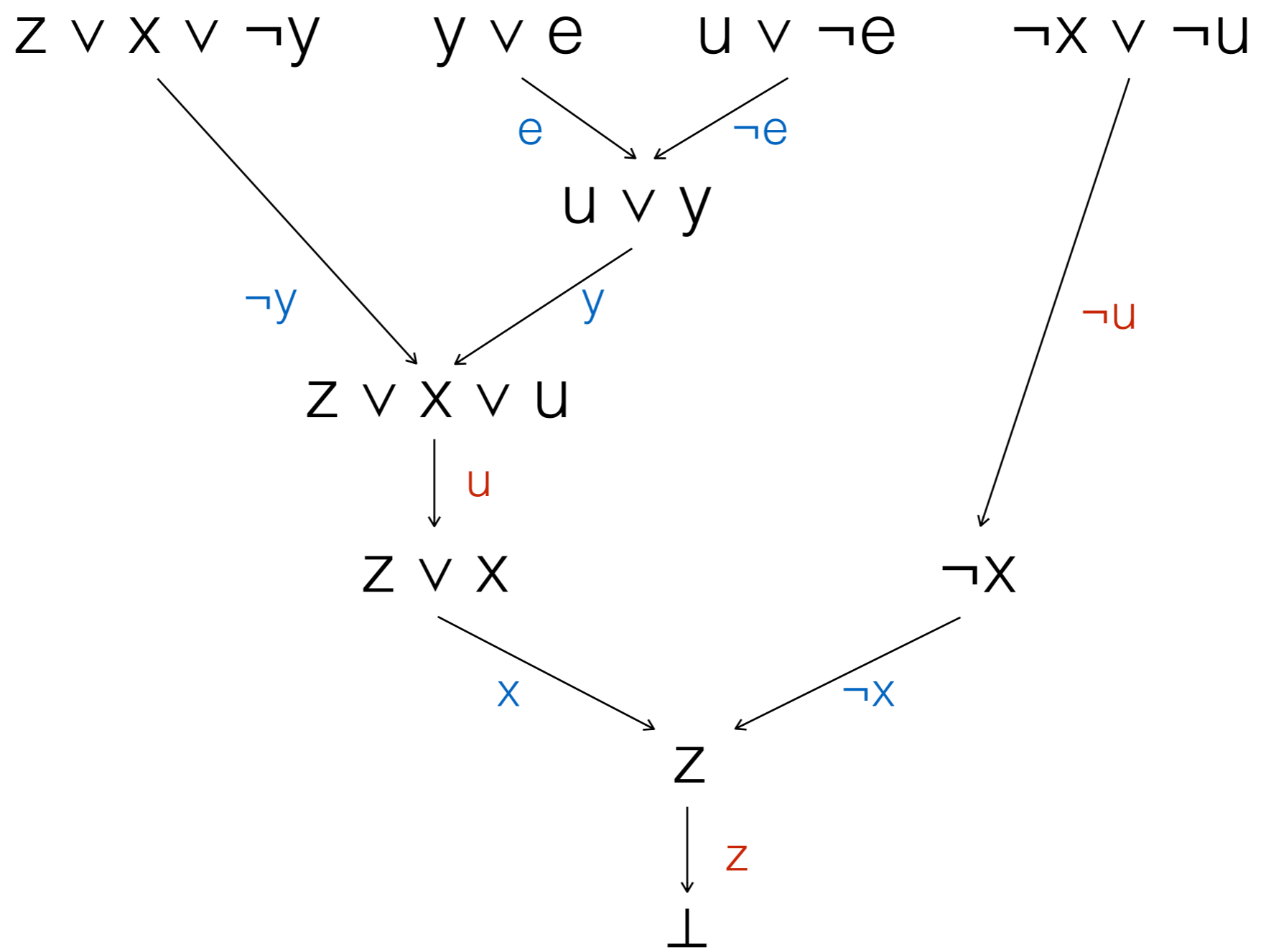
$\exists e \exists y \exists u \exists x \exists z$



$\exists e \forall u \exists x \forall z \exists y$



$\exists e \exists y \forall x \exists z$



Q(**D**)-resolution

$$\exists e \frac{C \vee e \quad \neg e \vee C'}{C \vee C'} \quad \text{resolution}$$

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$$\forall u \frac{C \vee u}{C}$$

D-reduction

Q(**D**)-resolution

$$\exists e \frac{C \vee e \quad \neg e \vee C'}{C \vee C'} \quad \text{resolution}$$

$$\forall u \frac{C \vee u}{C} \quad \mathbf{D}\text{-reduction}$$

every x in C is **independent** of u

Q(**D**)-resolution

$$\exists e \frac{C \vee e \quad \neg e \vee C'}{C \vee C'} \quad \text{resolution}$$

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every x in C is **independent** of u

D(**F**):
 u
 x

Q(**D**)-resolution

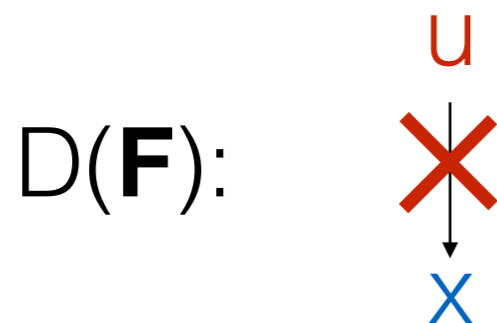
$$\exists e \frac{C \vee e \quad \neg e \vee C'}{C \vee C'}$$

resolution

$$\forall u \frac{C \vee u}{C}$$

D-reduction

every x in C is **independent** of u



$\exists x \forall u \exists y \exists e$

$u \vee y$

$\neg u \vee e$

$\neg e \vee x$

$\neg x \vee \neg y$

$\exists x \forall u \exists y \exists e$

$u \vee y$

$\neg u \vee e$

$\neg e \vee x$

$\neg x \vee \neg y$



u

y

$\exists x \forall u \exists y \exists e$

$u \vee y$

$\neg u \vee e$

$\neg e \vee x$

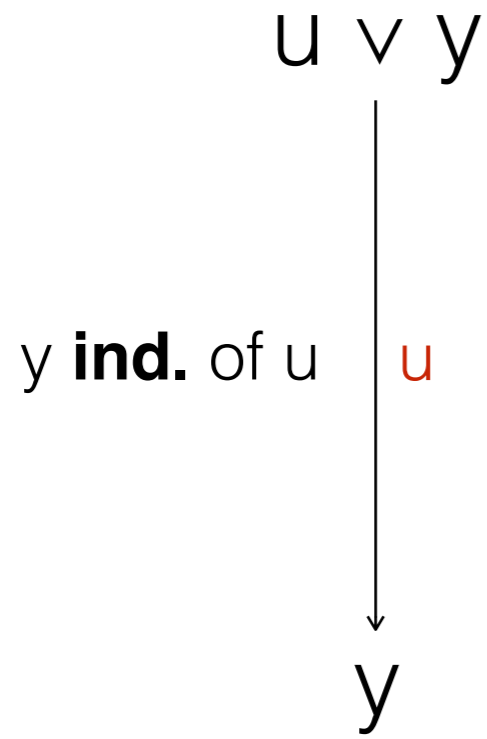
$\neg x \vee \neg y$

y **ind.** of u

u

y

$\exists x \forall u \exists y \exists e$



$\neg u \vee e$

$\neg e \vee x$

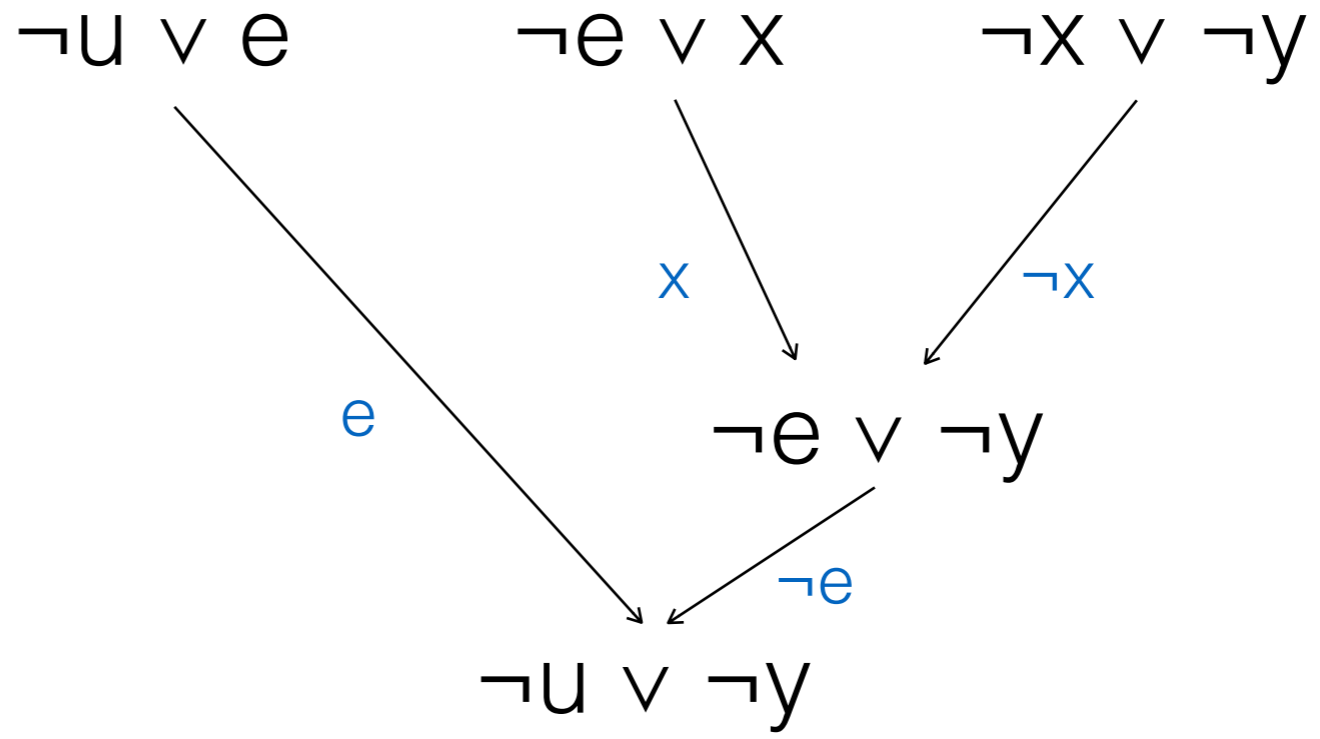
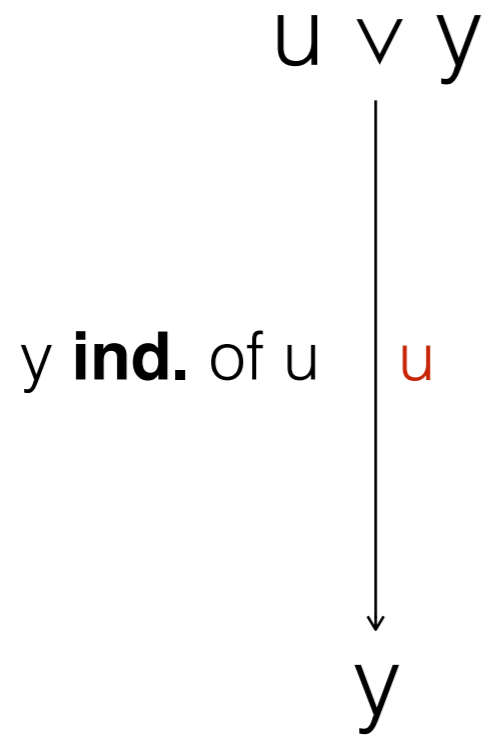
$\neg x \vee \neg y$

x

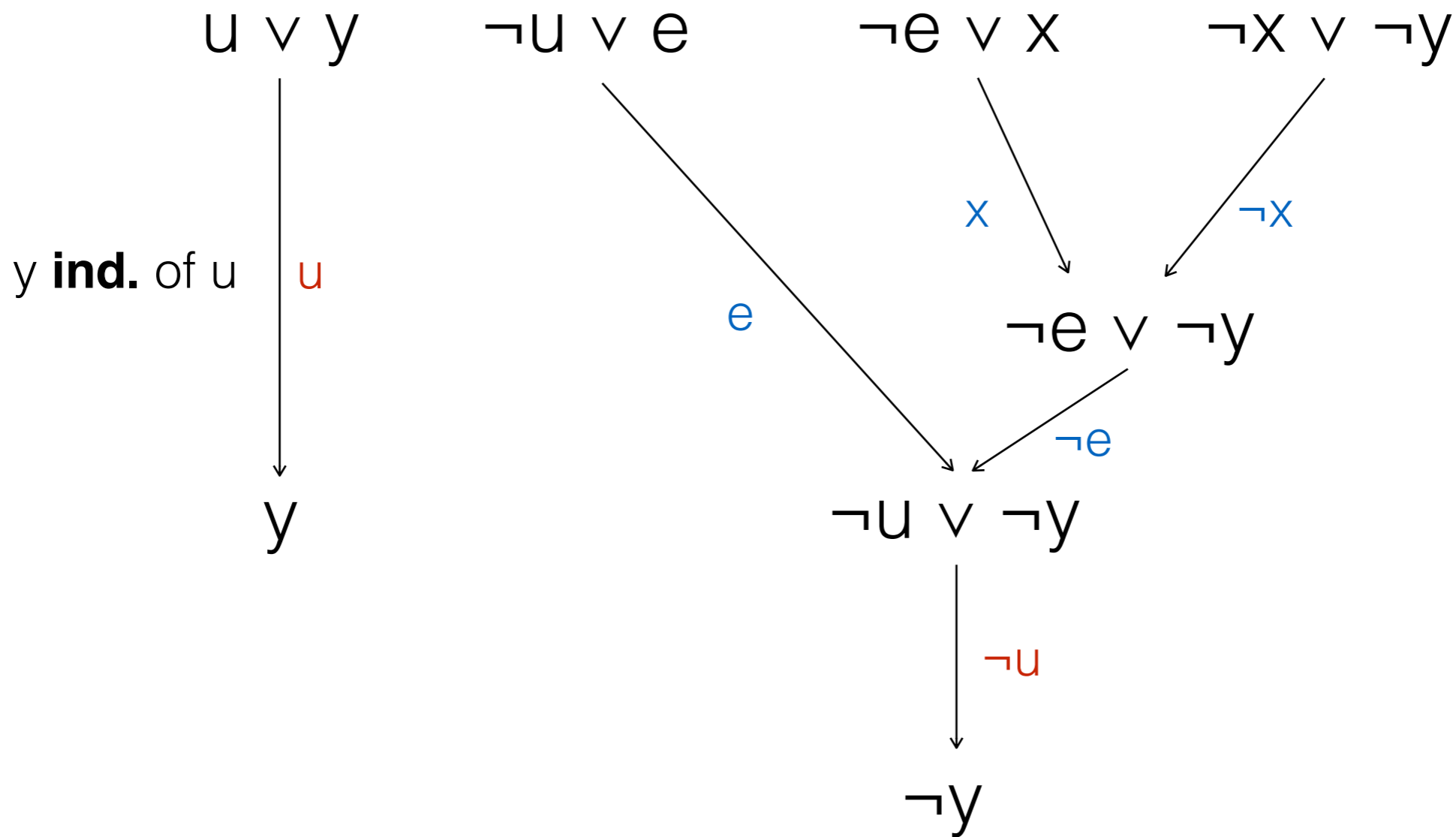
$\neg x$

$\neg e \vee \neg y$

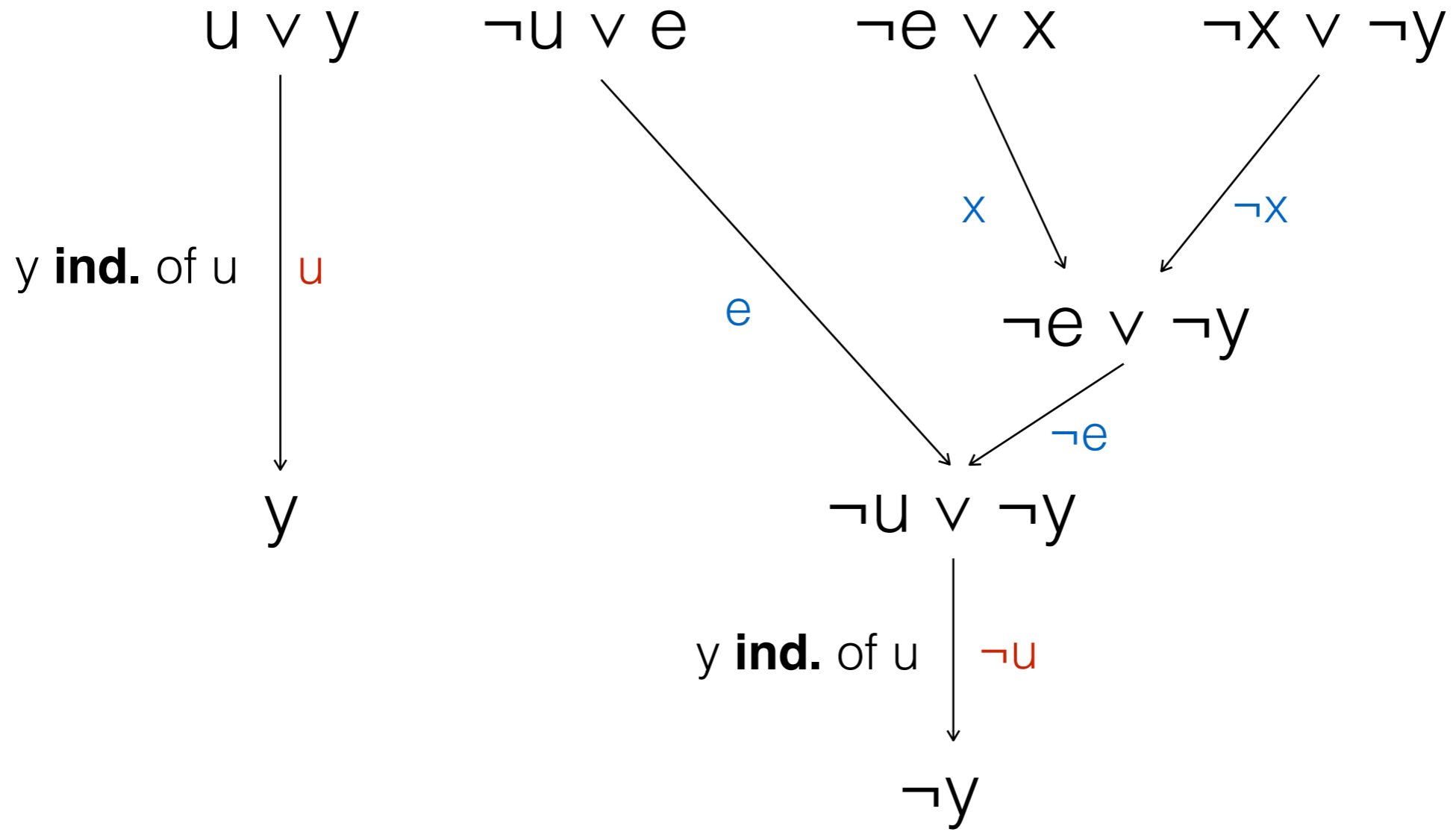
$\exists x \forall u \exists y \exists e$



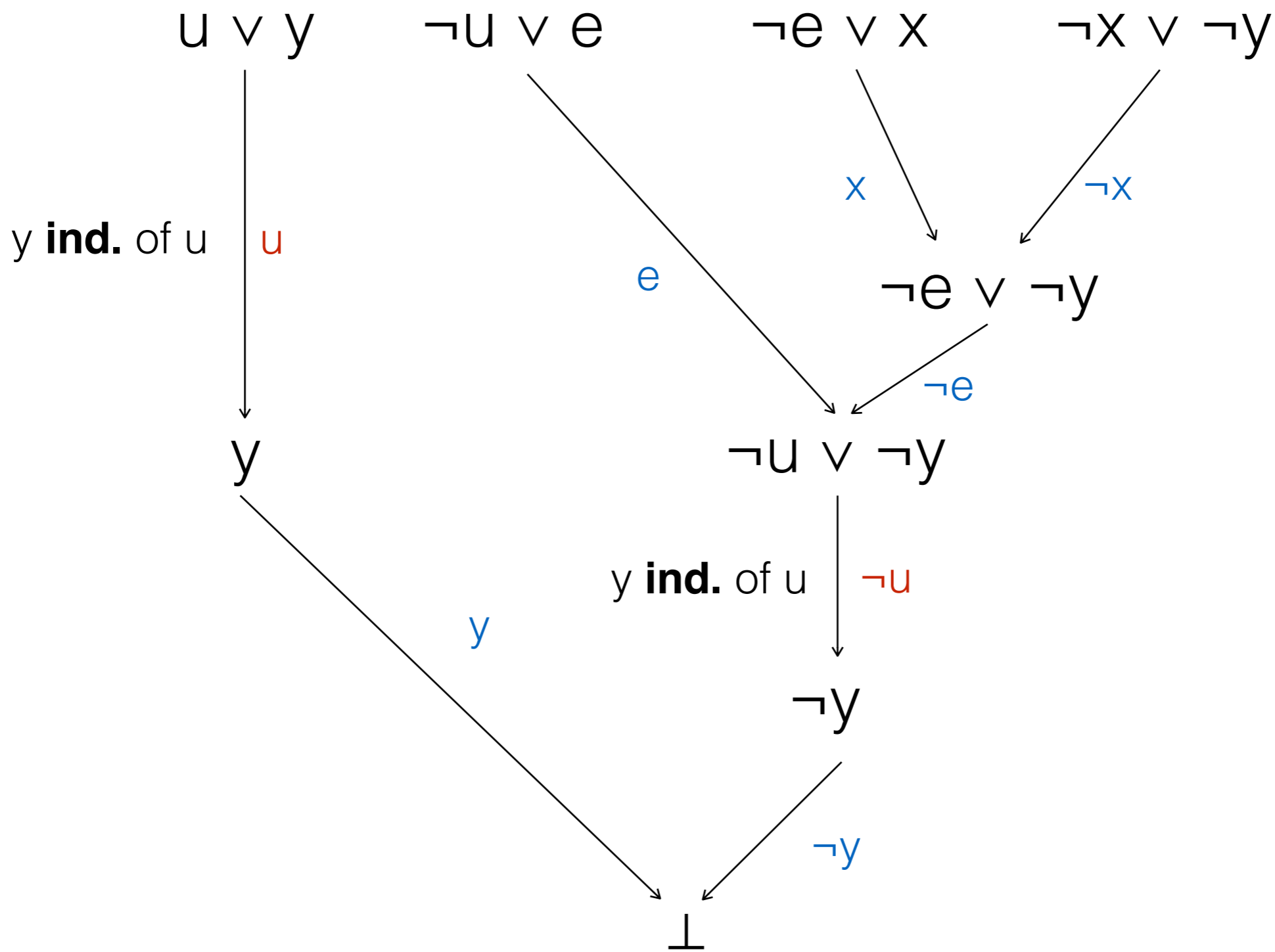
$\exists x \forall u \exists y \exists e$



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$\exists x \forall u \exists y \exists e$



Q(Drrs)-resolution is sound.

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Proof strategy

Q(Drrs)-resolution is sound.

Proof strategy

Q(Drrs)-resolution refutation of **F**

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Proof strategy

Q(Drrs)-resolution refutation of **F**



rewriting

Q(Drrs)-resolution is sound.

Proof strategy

Q(Drrs)-resolution refutation of **F**

rewriting



Q-resolution refutation of **F**

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Q(Drrs)-resolution refutation of **F**

D-reduction

rewriting

Q-resolution refutation of **F**

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Proof strategy

Q(Drrs)-resolution refutation of **F**

D-reduction

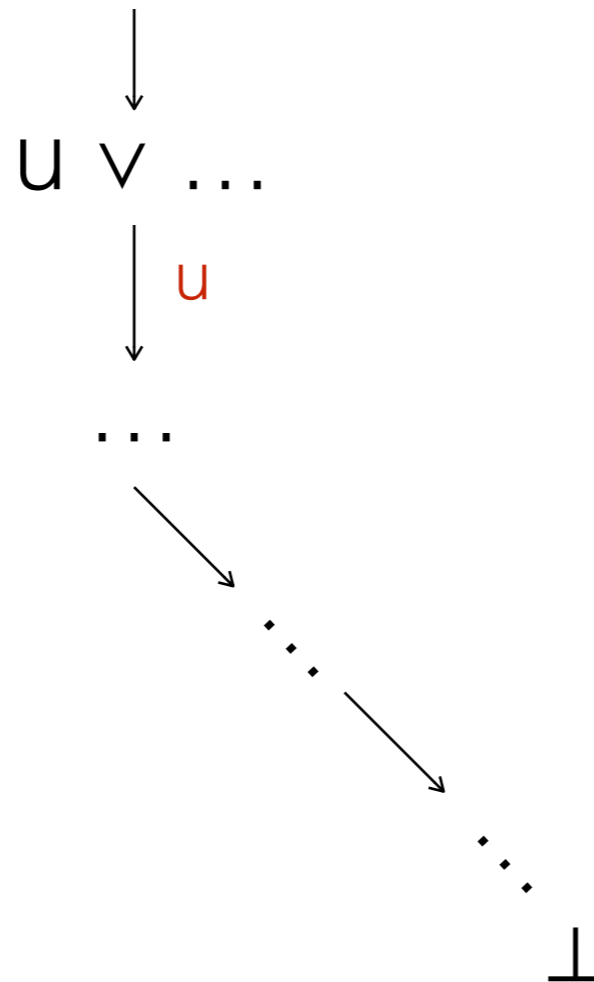
rewriting

universal
reduction

Q-resolution refutation of **F**

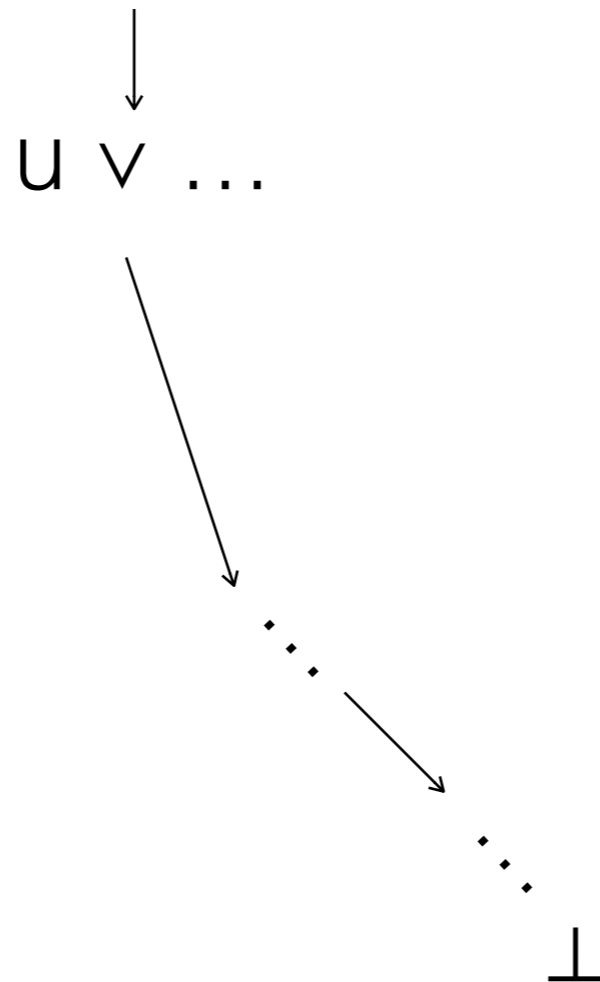
proof sketch

delaying D-reductions



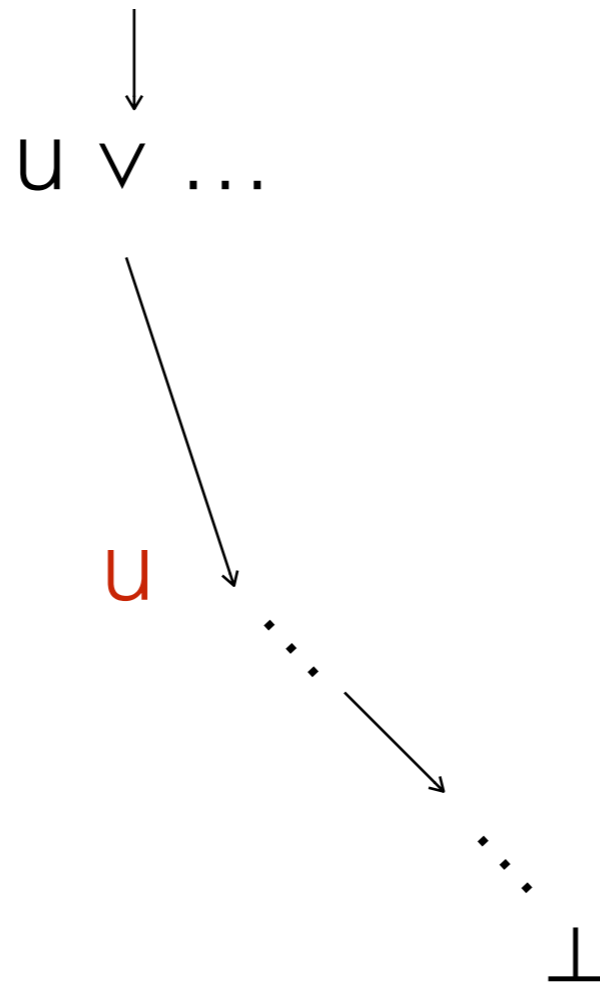
proof sketch

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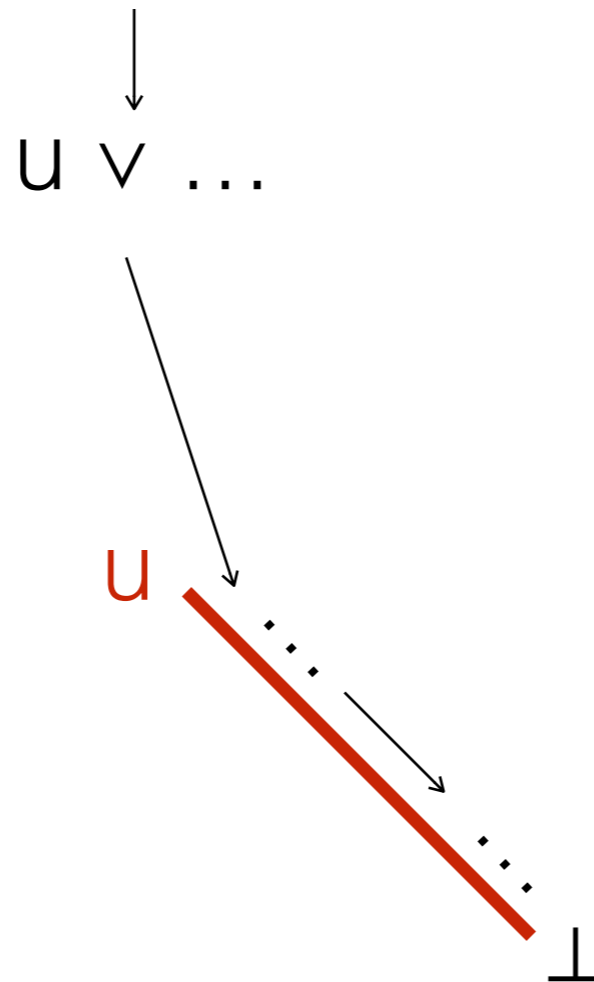
proof sketch

delaying D-reductions



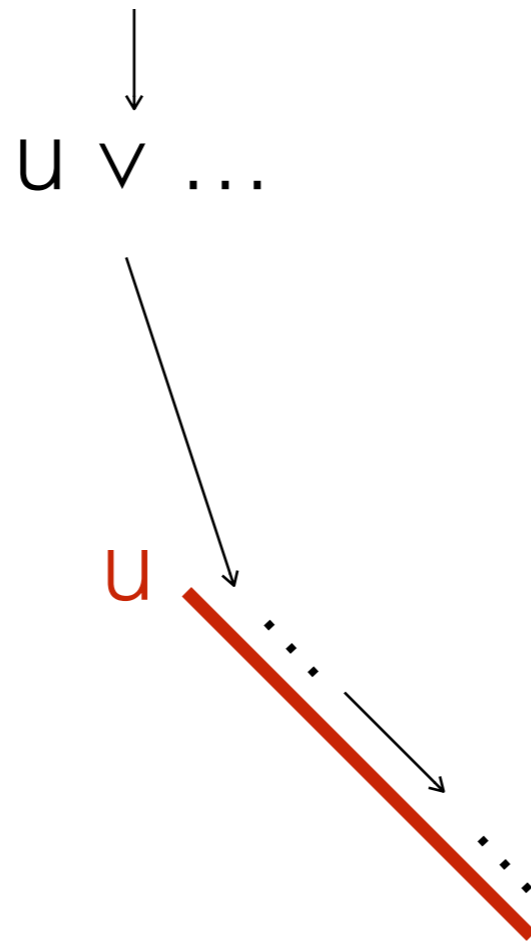
proof sketch

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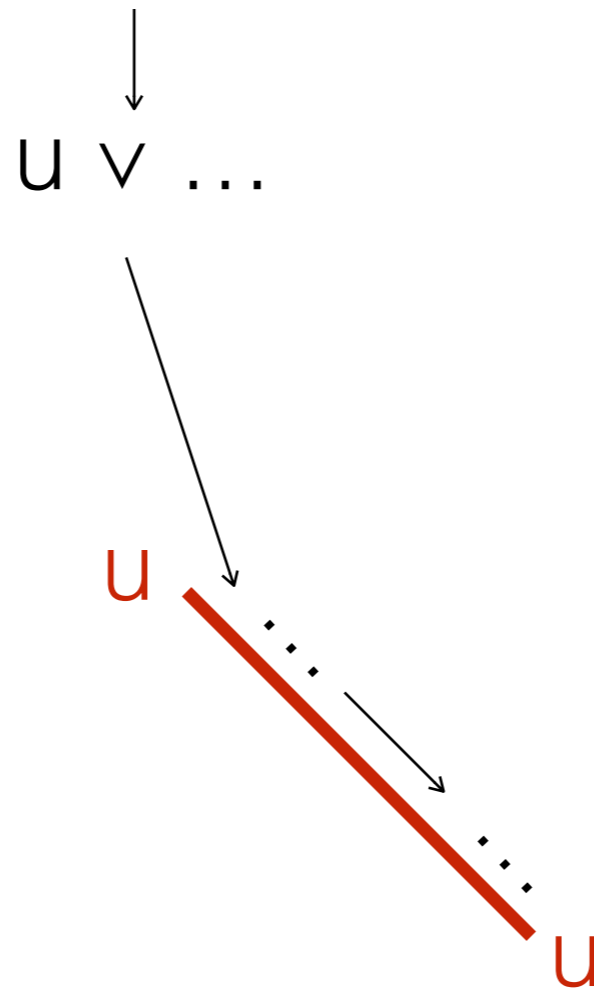
proof sketch

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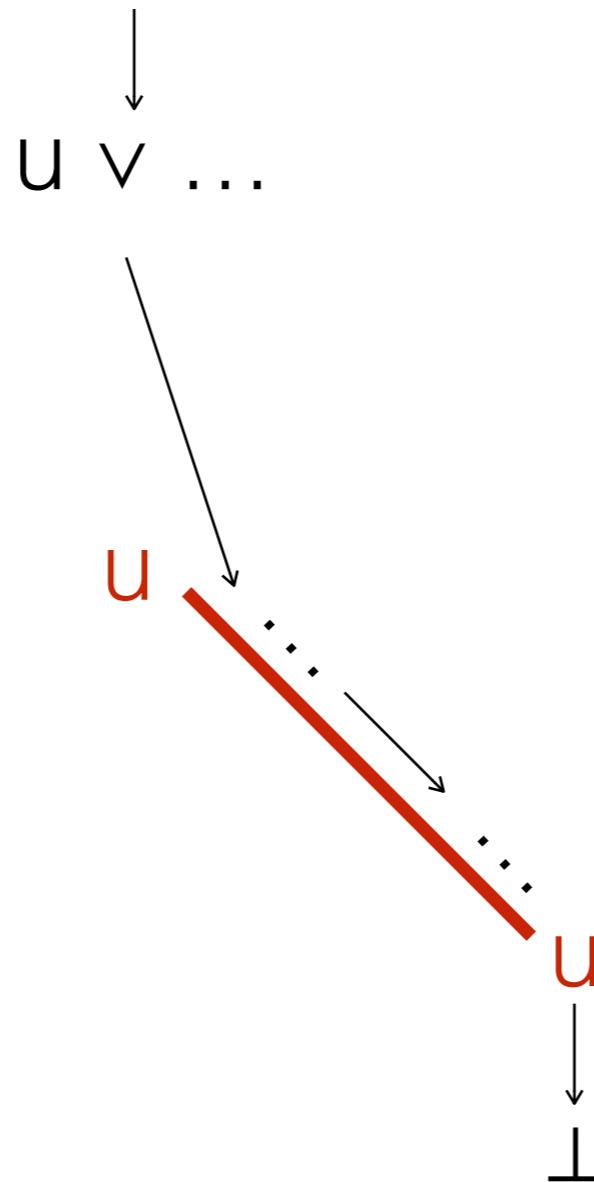
proof sketch

delaying D-reductions



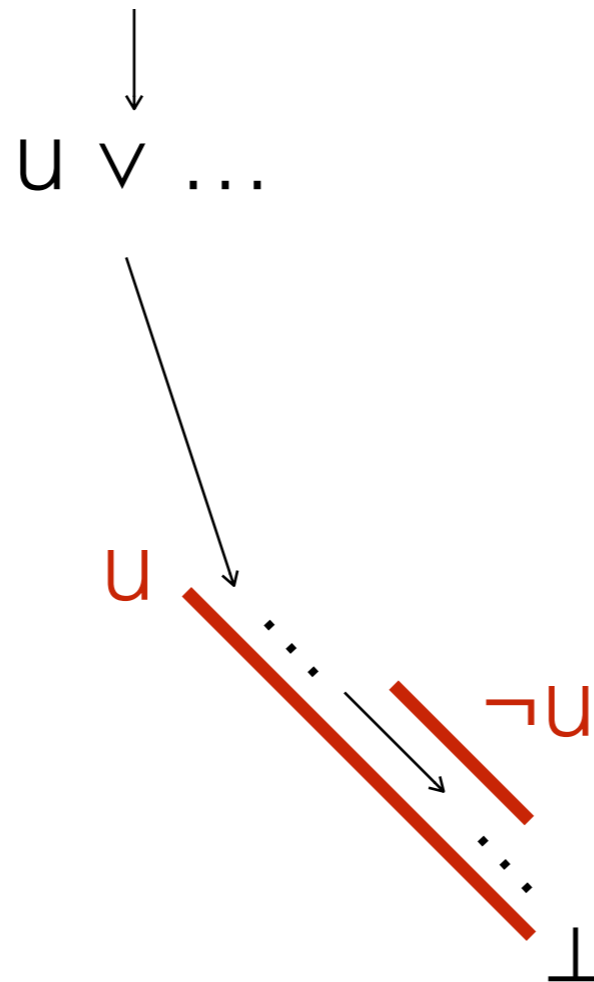
proof sketch

delaying D-reductions



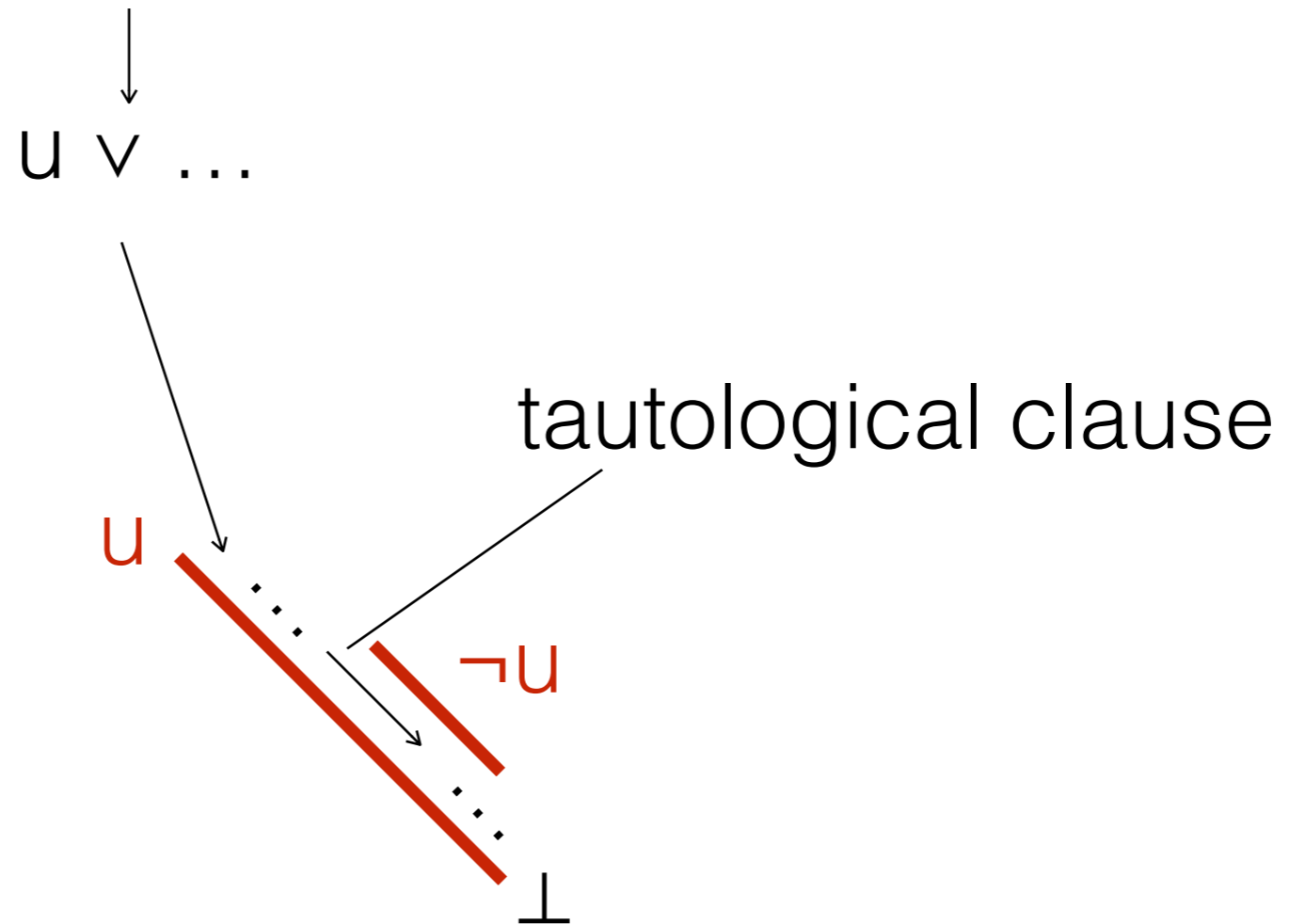
proof sketch

delaying D-reductions

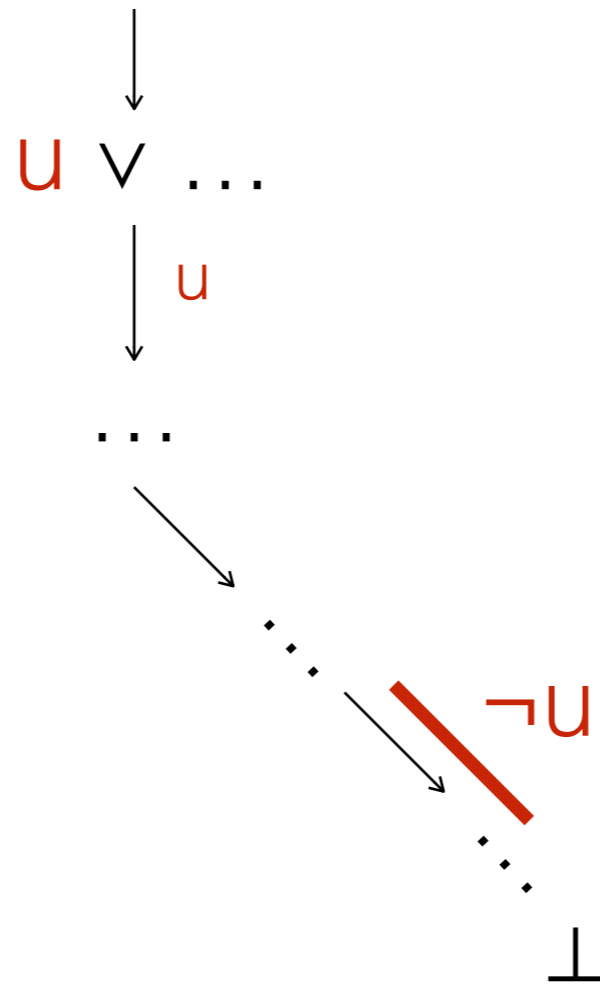


proof sketch

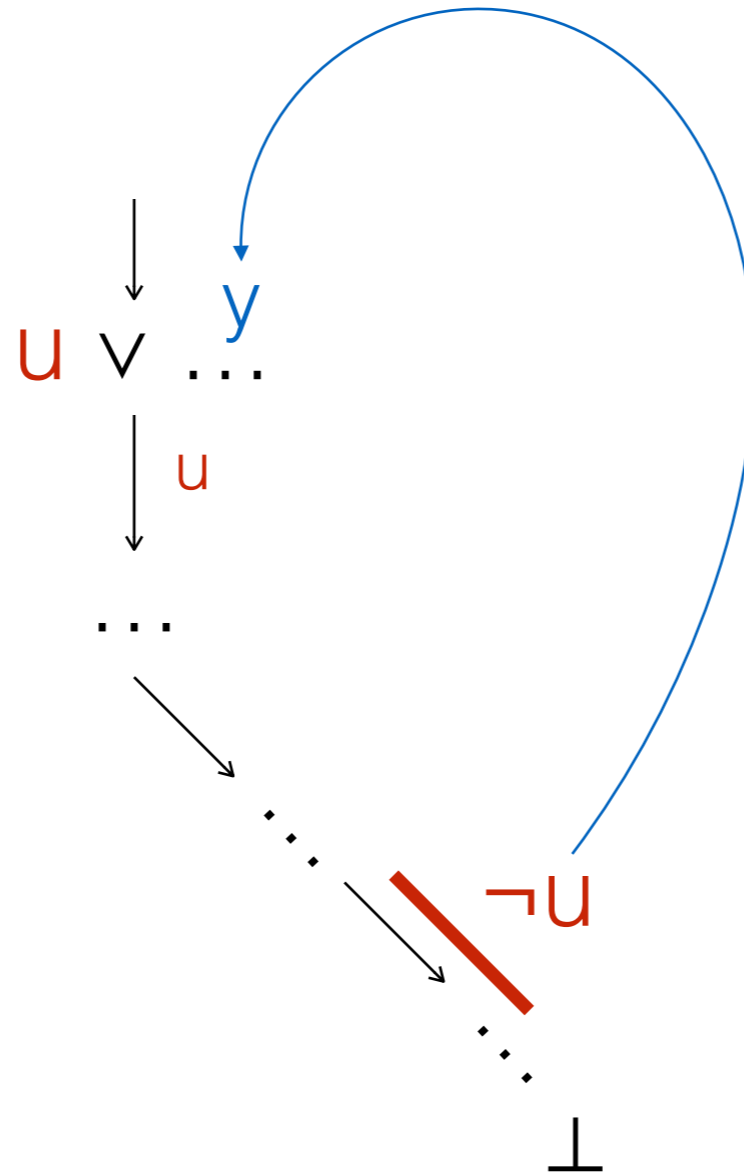
delaying D-reductions



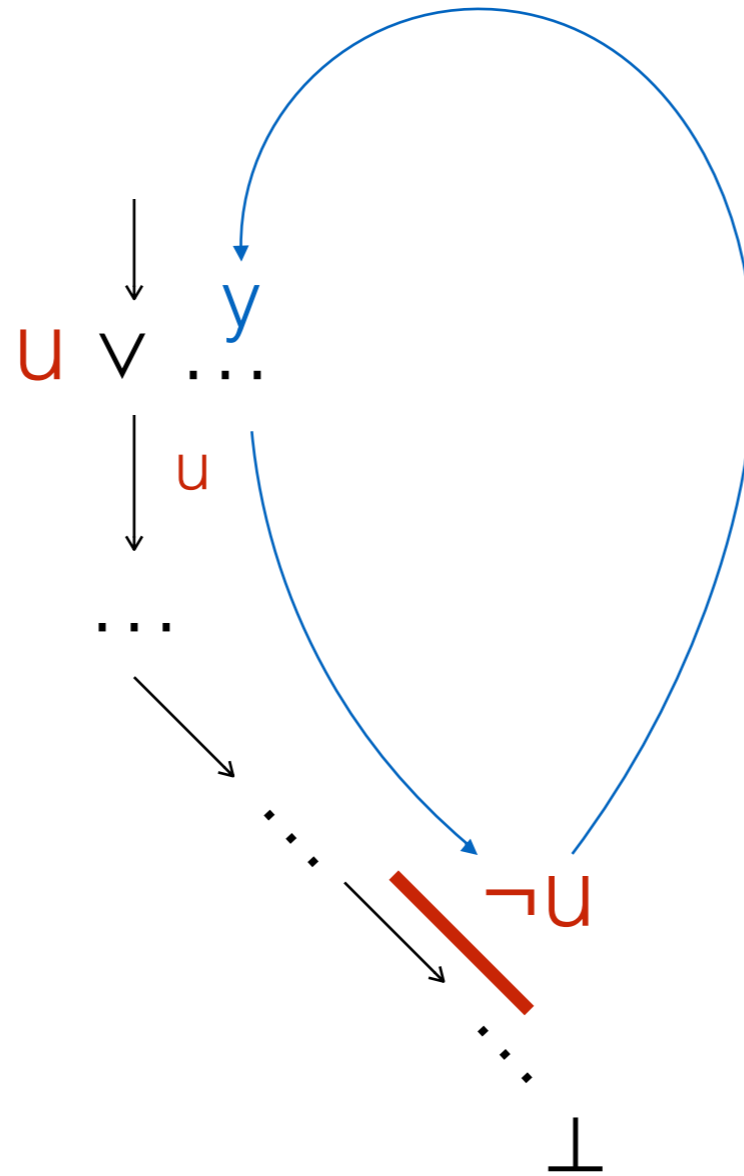
proof sketch



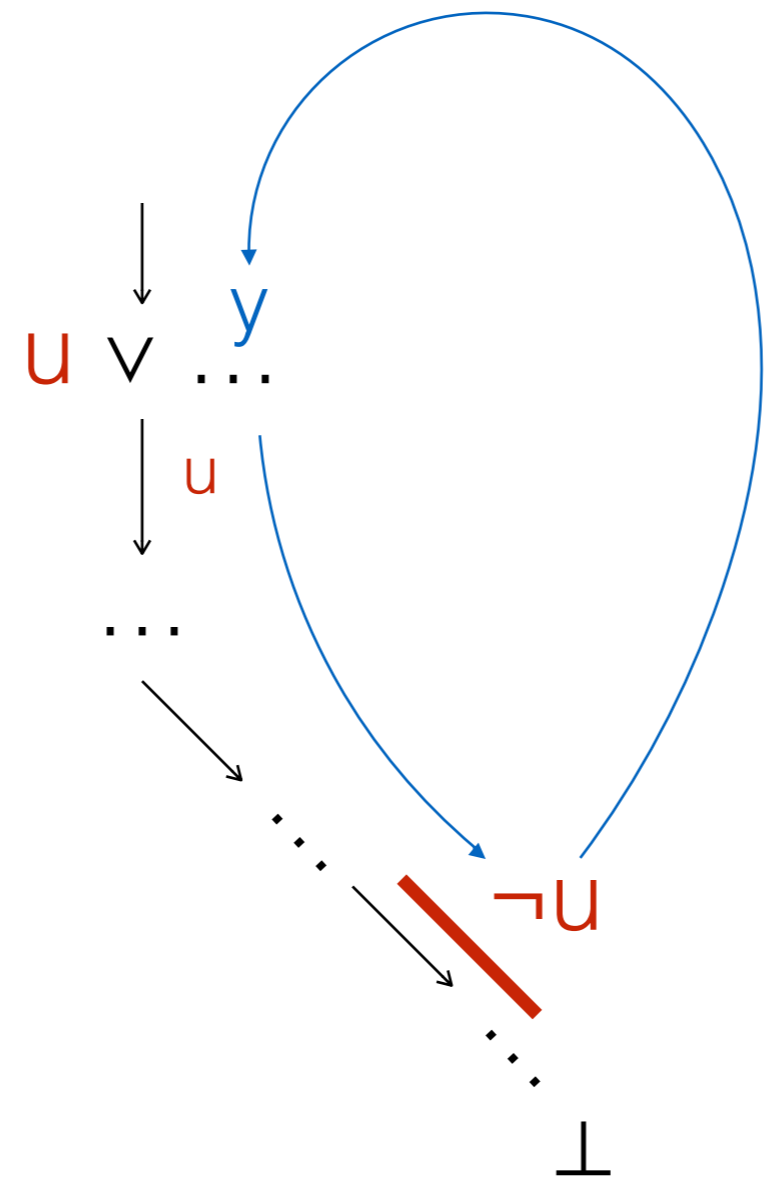
proof sketch



proof sketch

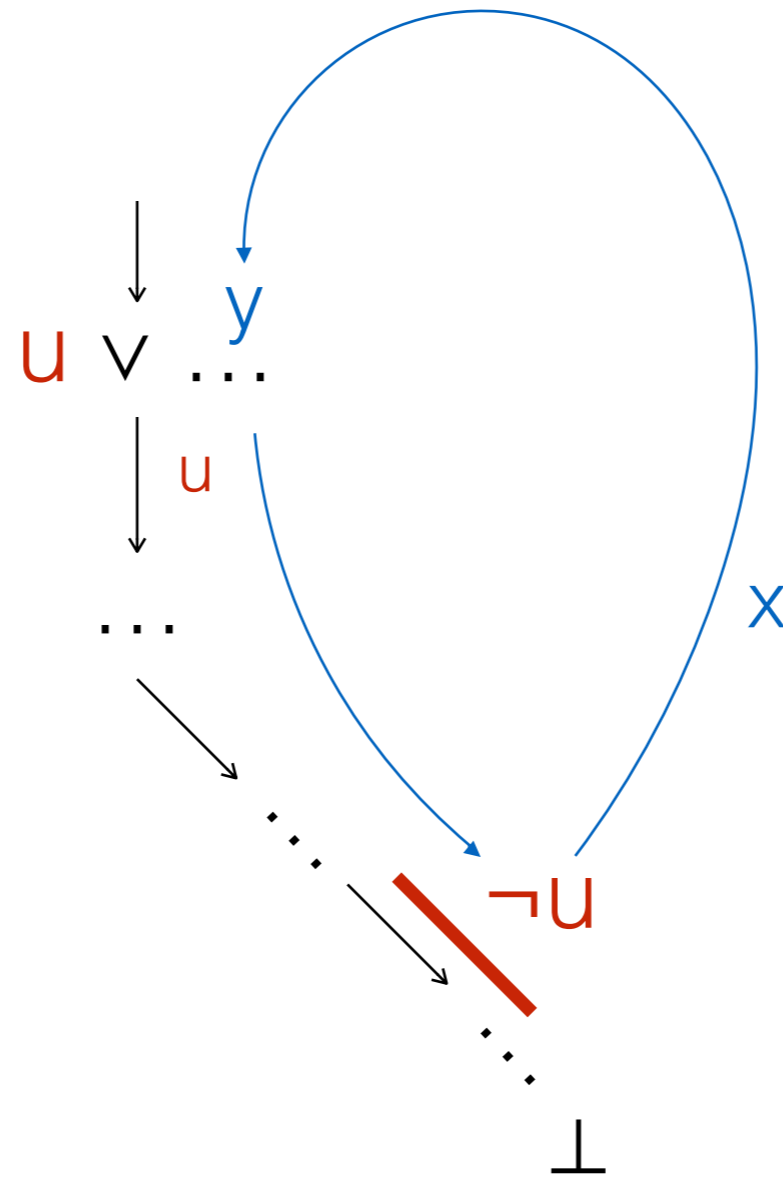


proof sketch



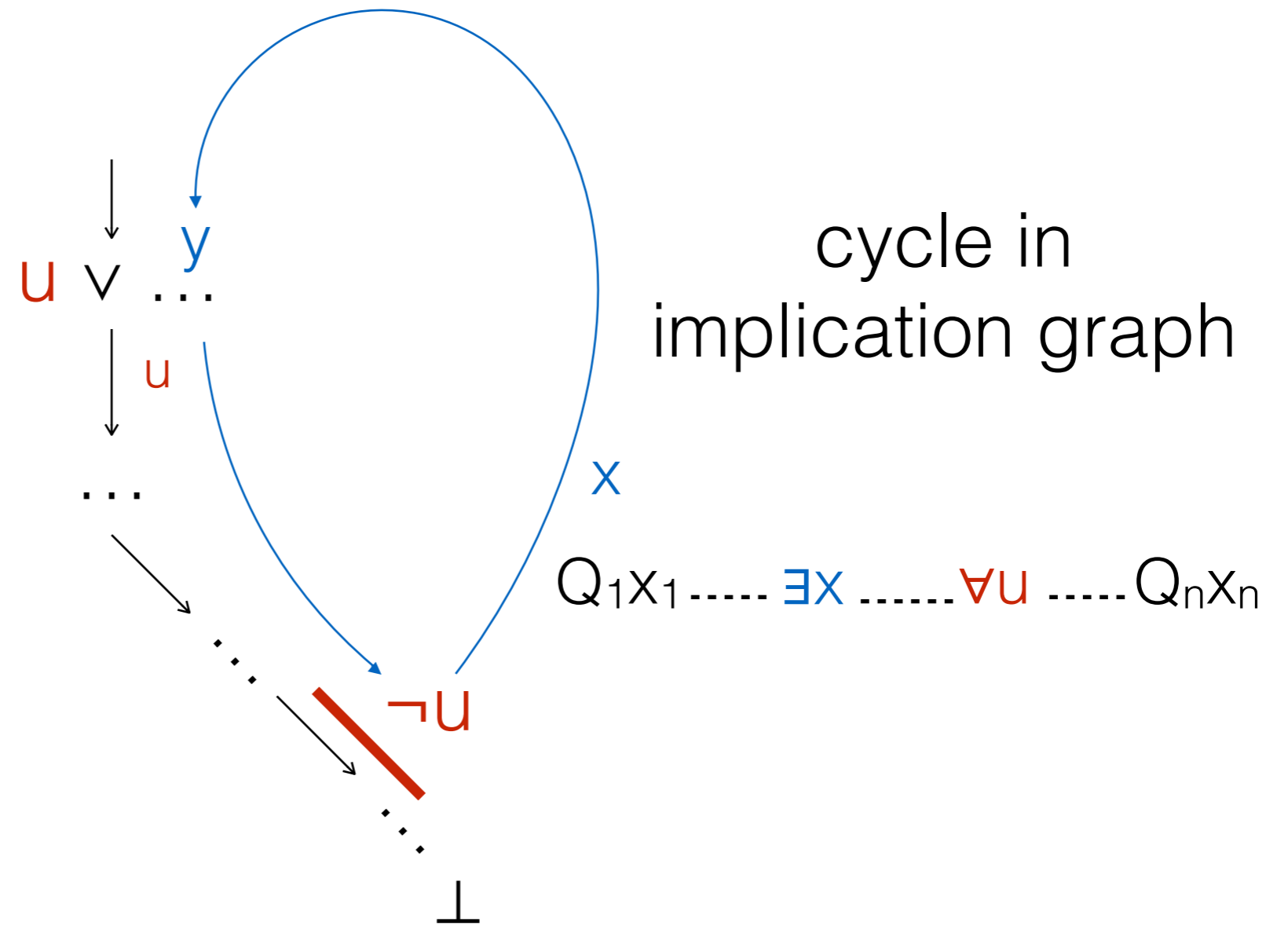
cycle in
implication graph

proof sketch

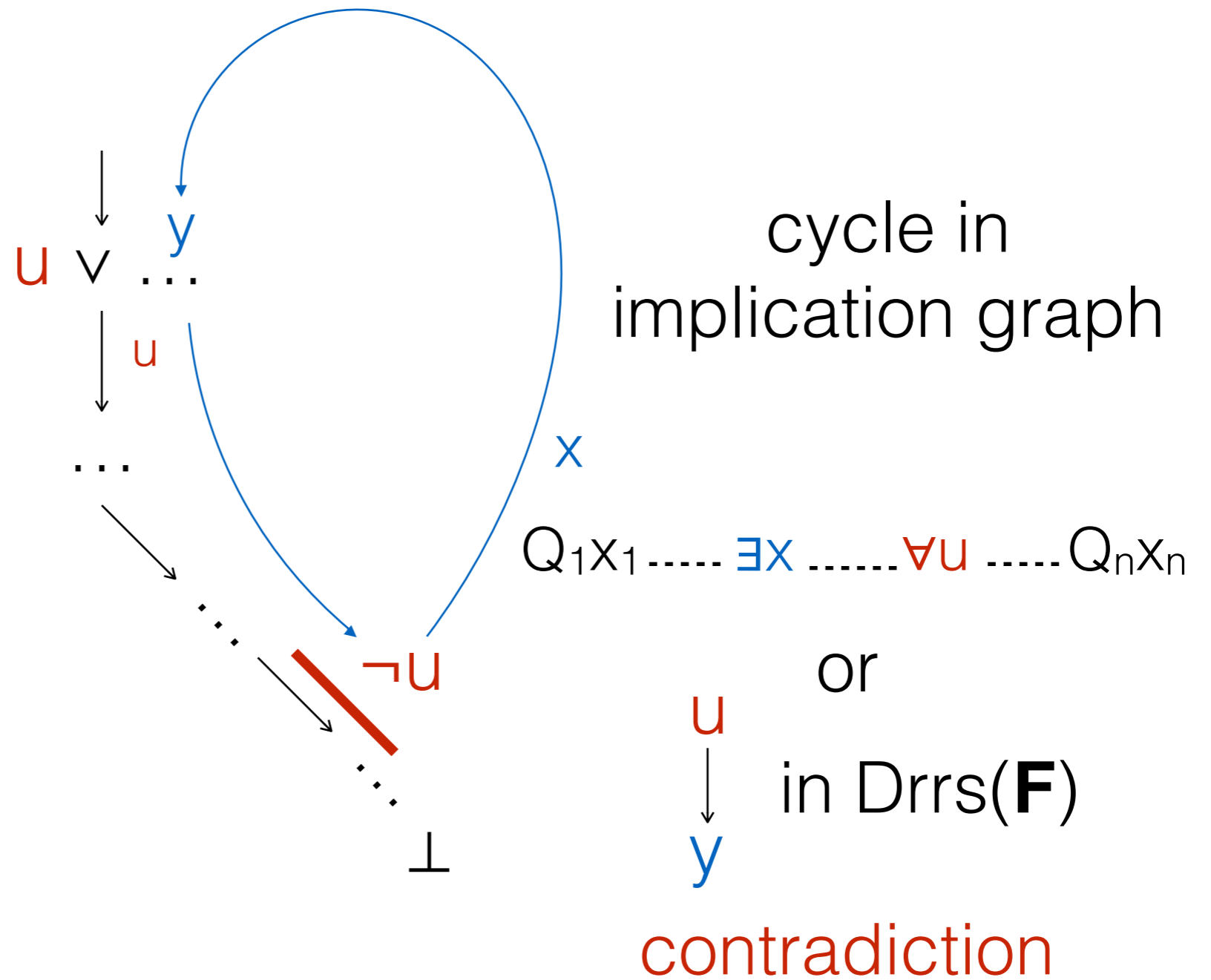


cycle in
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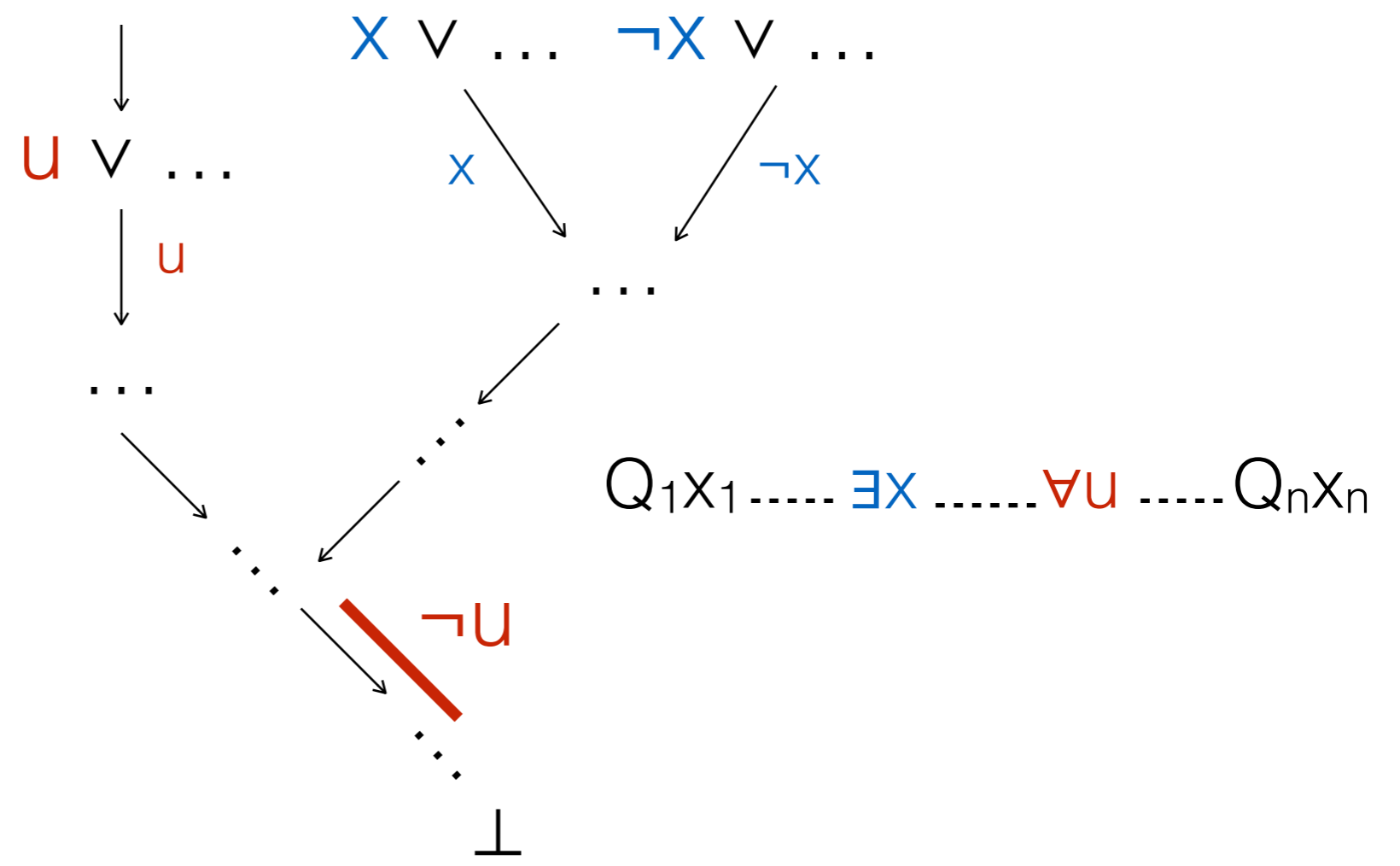
proof sketch



proof sketch

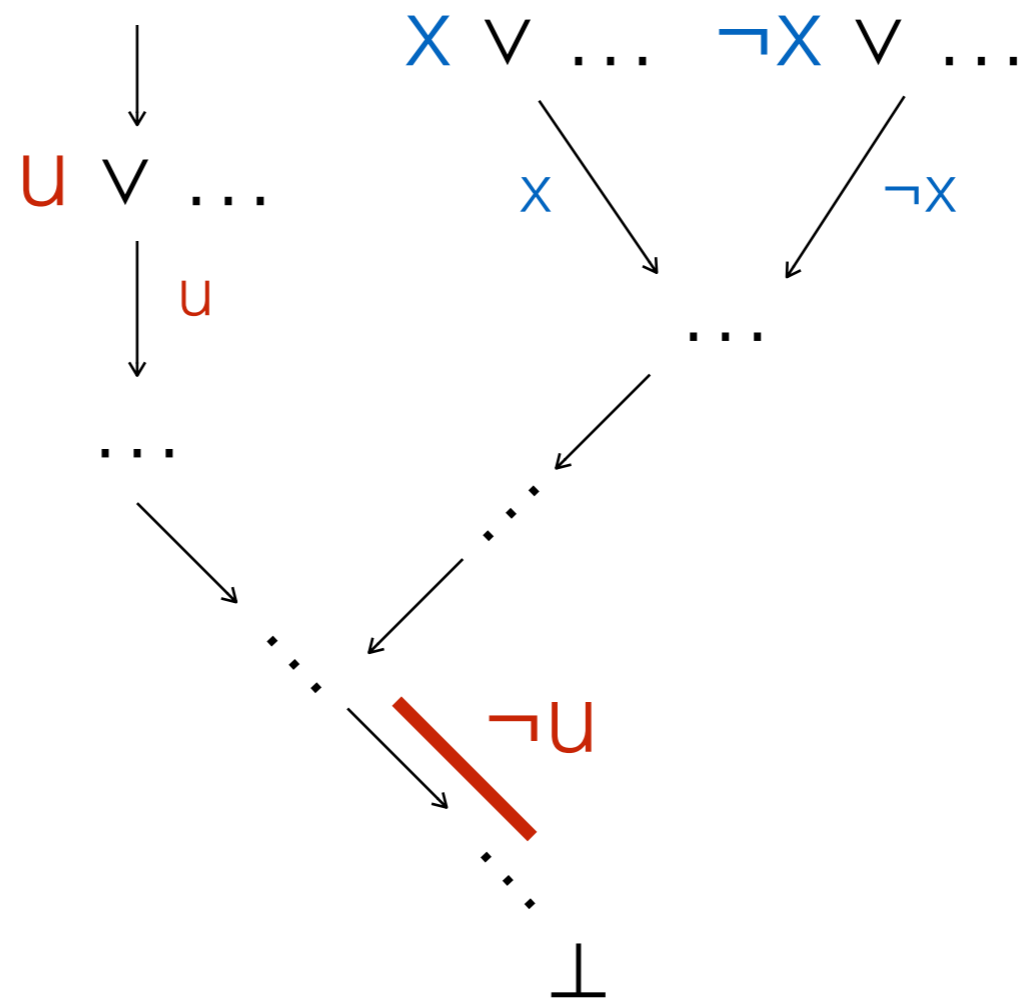


proof sketch



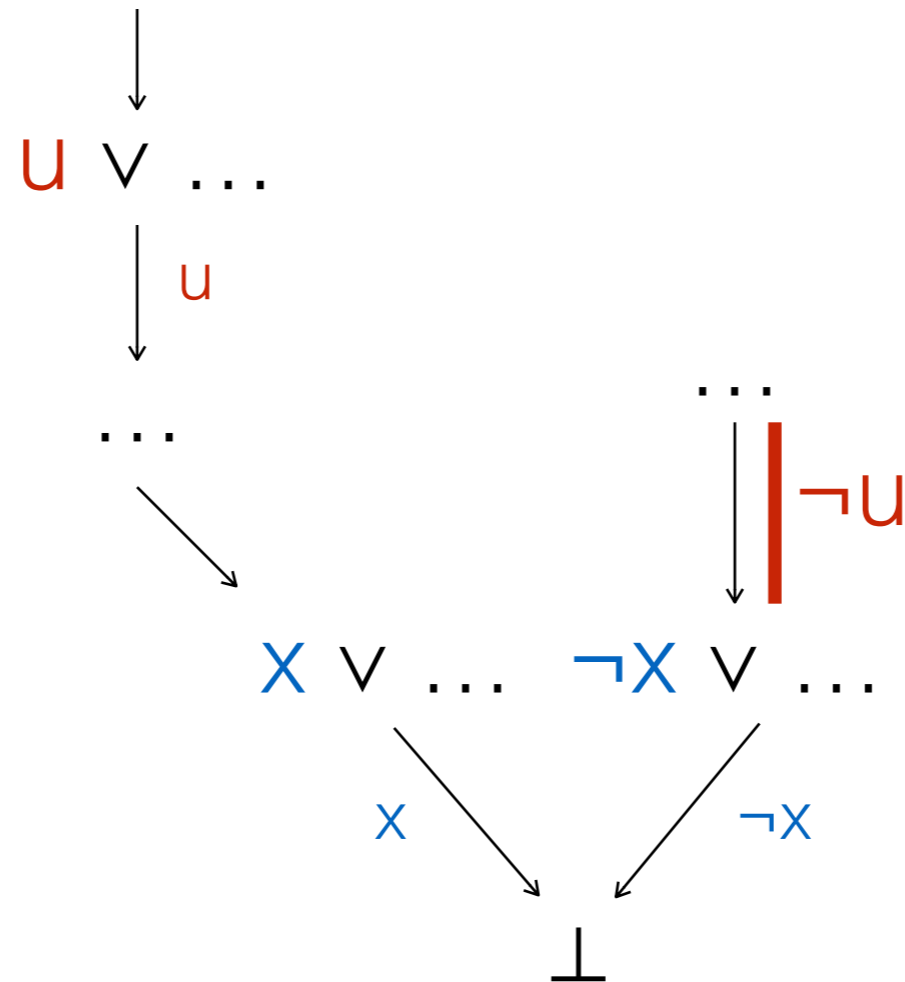
proof sketch

lower resolution



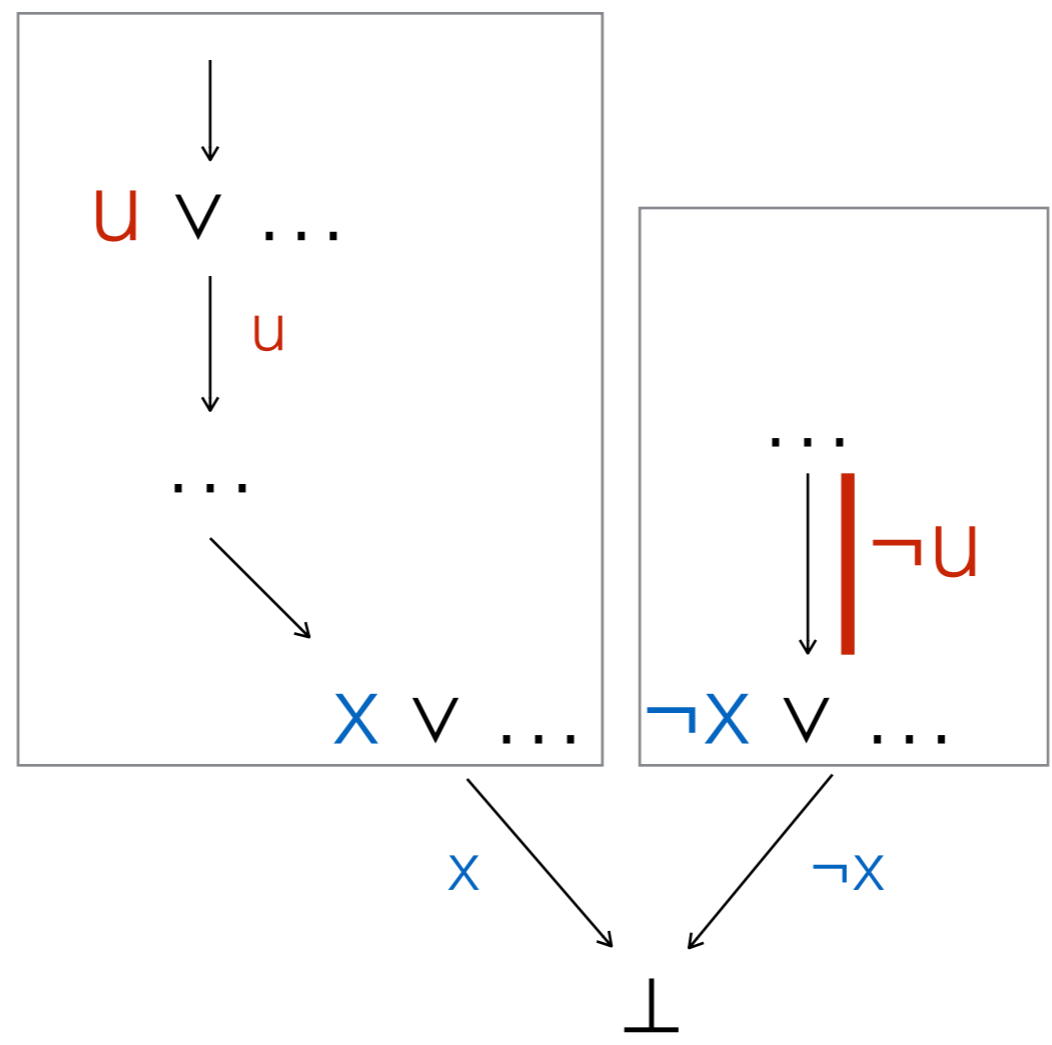
proof sketch

lower resolution



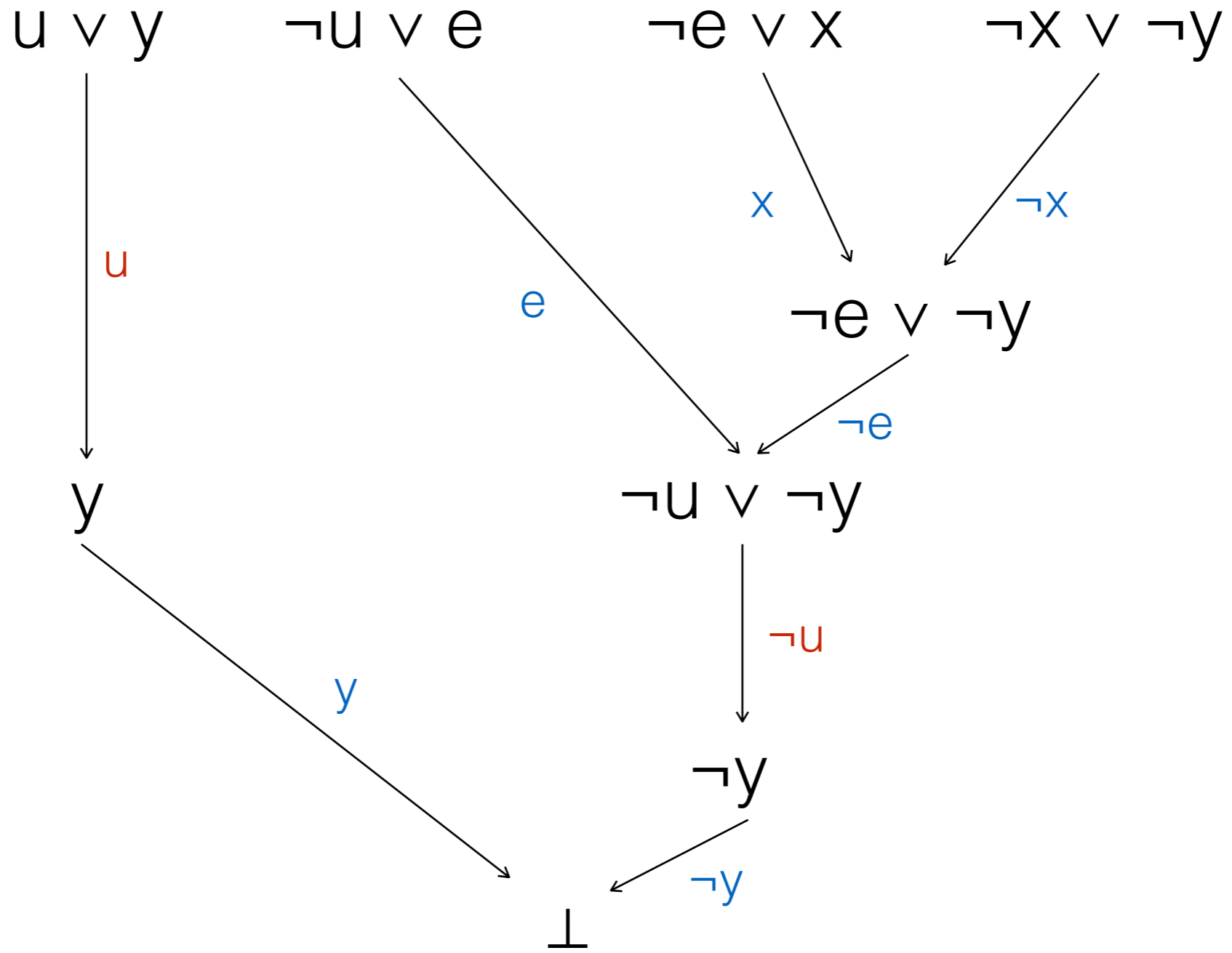
proof sketch

recurse on subderivations



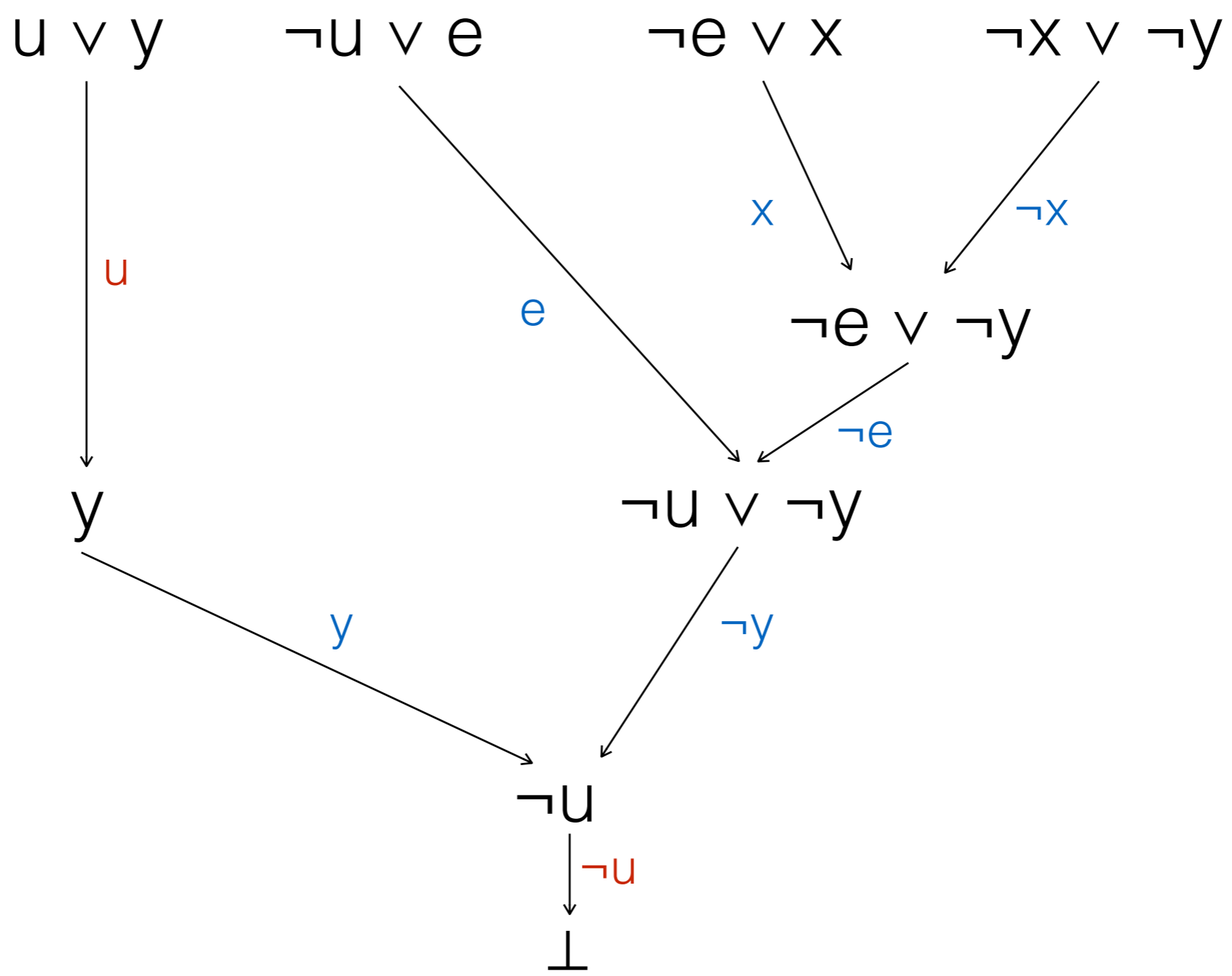
proof by example

$\exists x \forall u \exists y \exists e$



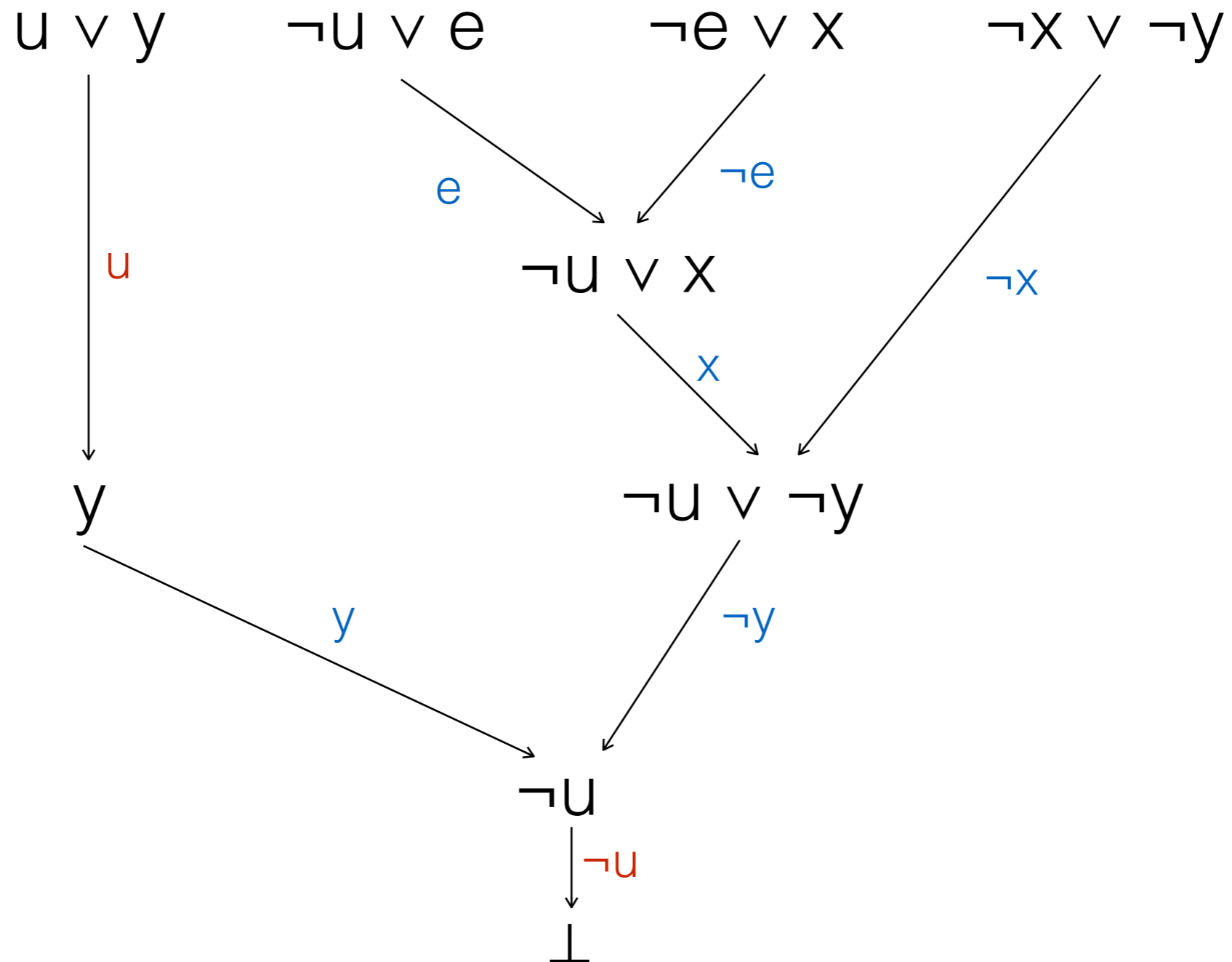
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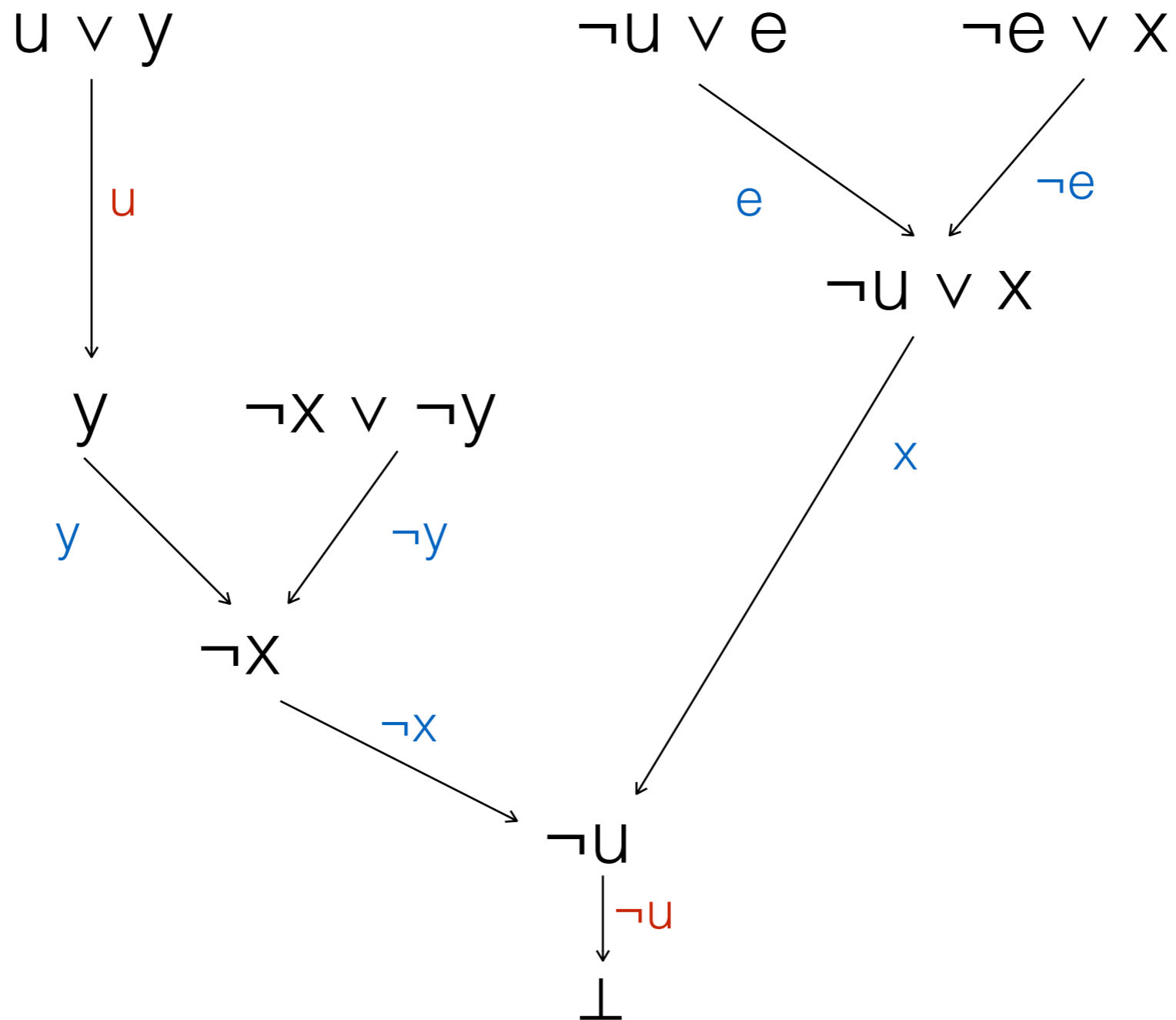
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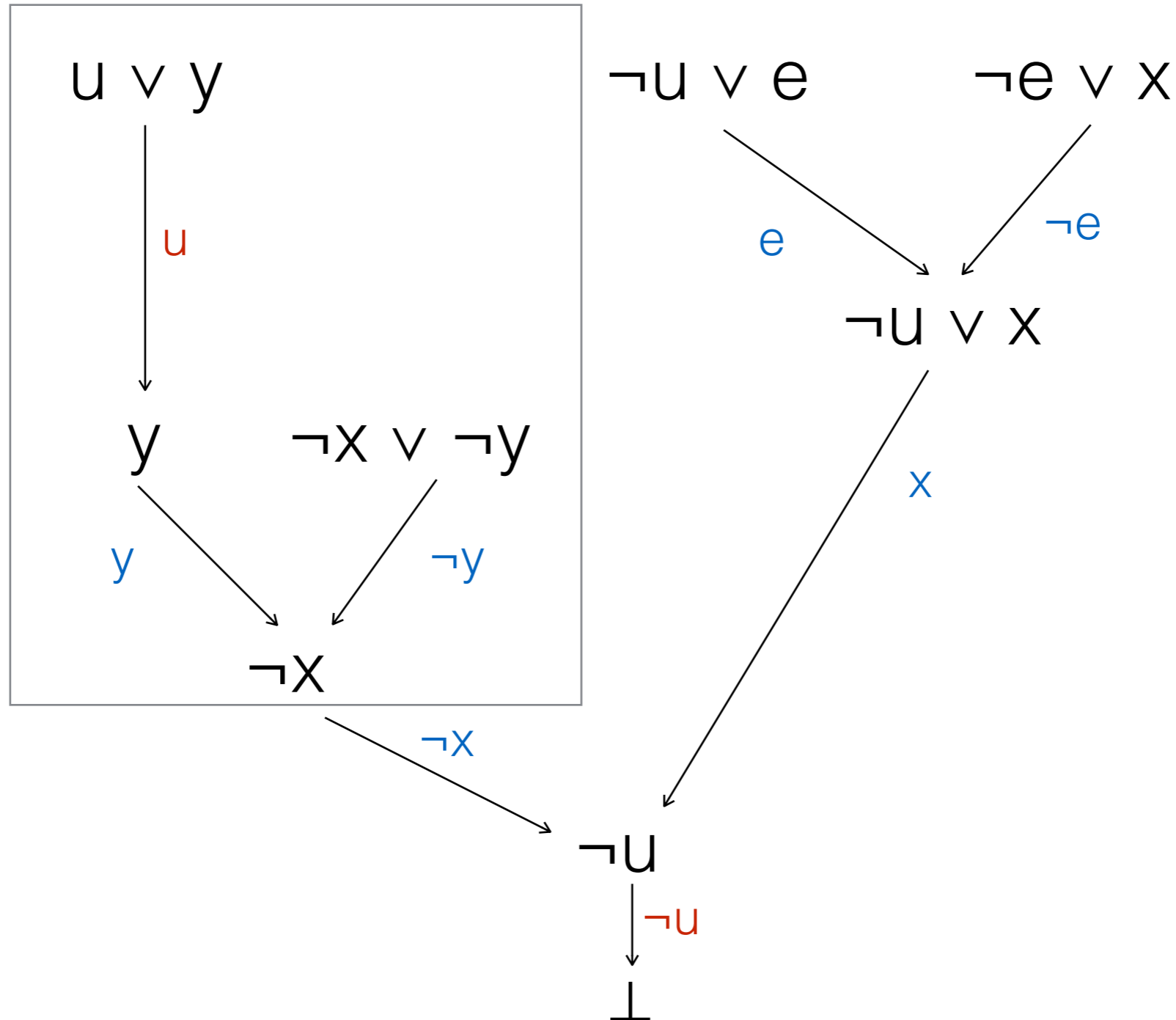
proof by example

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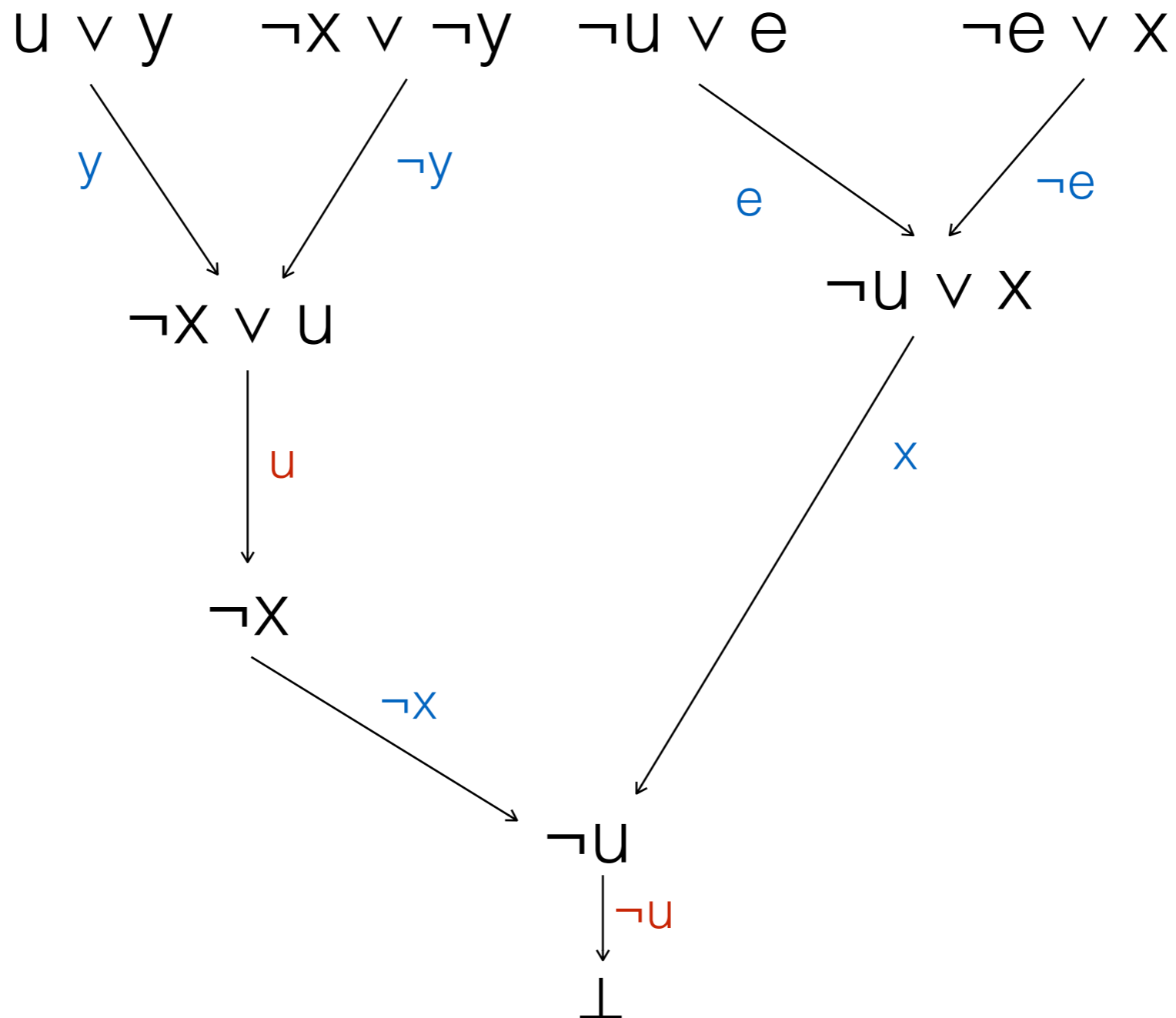
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proof by example

$\exists x \forall u \exists y \exists e$



open problems

open problems

dependency schemes for [universal expansion](#)

open problems

dependency schemes for [universal expansion](#)

[fast certificate extraction](#) from Q(D)-resolution proofs

open problems

dependency schemes for [universal expansion](#)

[fast certificate extraction](#) from Q(D)-resolution proofs

[relative complexity](#) of Q(D)-resolution proof systems