Fast DQBF Refutation

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Bernd Finkbeiner
Leander Tentrup

Reactive Systems Group
Saarland University

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DQBF

Dependency Quantified Boolean Formula

$$\forall x_1, x_2, x_3 \ldots \quad \exists_{H_1} y_1. \quad \exists_{H_2} y_2. \quad \exists_{H_3} y_3 \ldots \quad \varphi$$

DQBF

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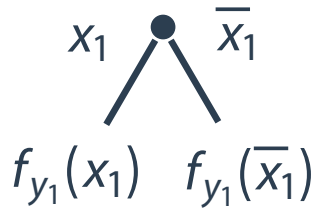
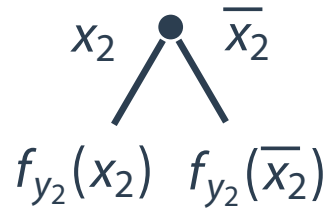
Example

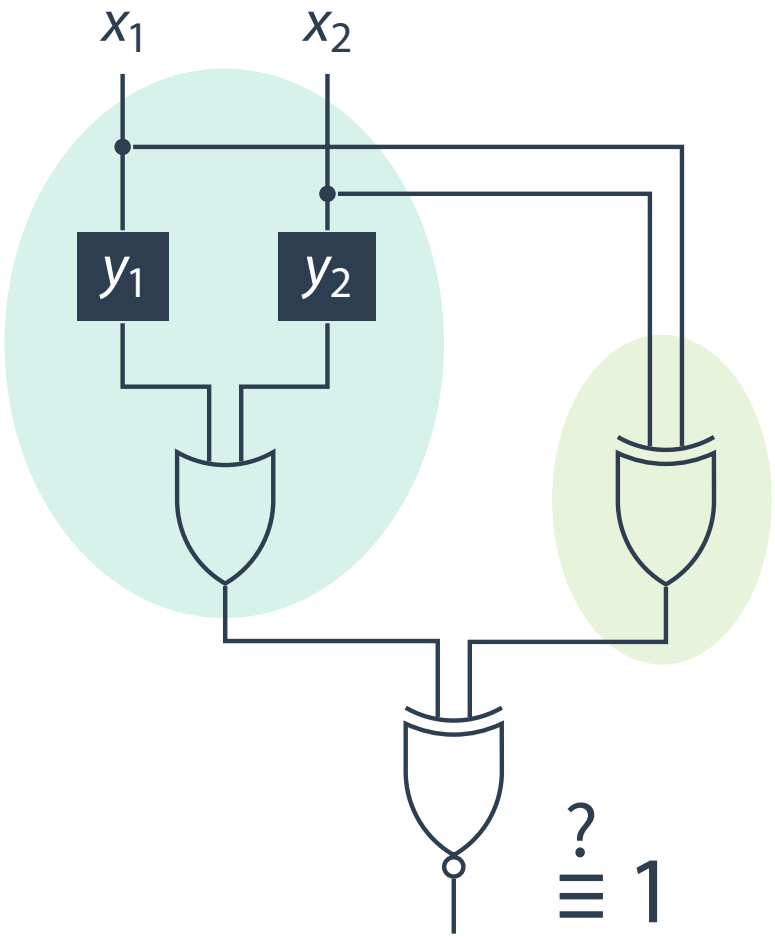
$\forall x_1, x_2. \exists_{\{x_1\}} y_1. \exists_{\{x_2\}} y_2. \varphi$





Partial Equivalence Checking (PEC)[1]

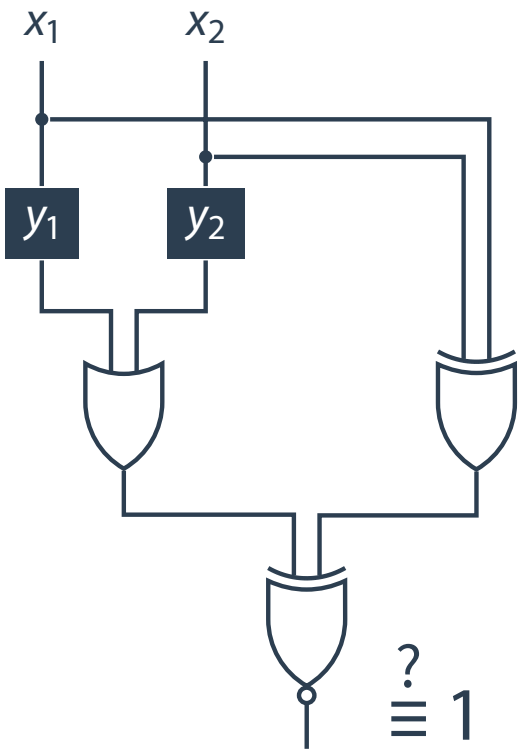
Partial Circuit



Specification

[1] Christoph Scholl and Bernd Becker. "Checking Equivalence for Partial Implementations". In: DAC '01. New York, NY, USA: ACM, 2001, pp. 238–243

Encoding of PEC in DQBF



$$\Leftrightarrow \quad \forall x_1, x_2.\, \exists_{\{x_1\}} y_1.\, \exists_{\{x_2\}} y_2.\, (y_1 \vee y_2) \leftrightarrow (x_1 \otimes x_2)$$

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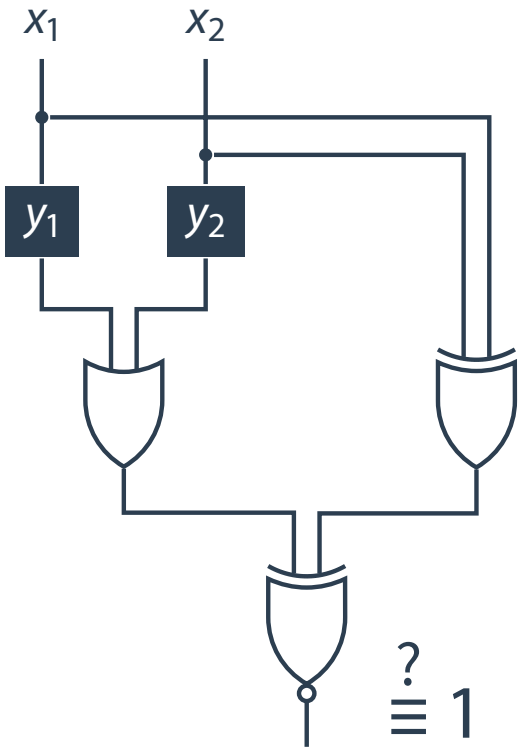

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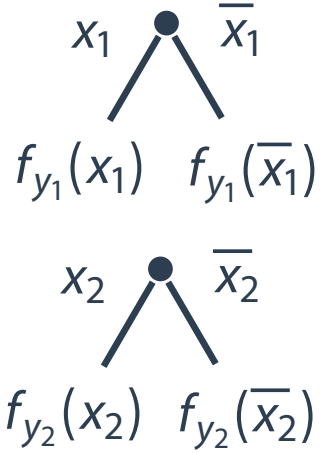
This paper: **Focus on refutation ⇒ Detect errors in PEC instances**

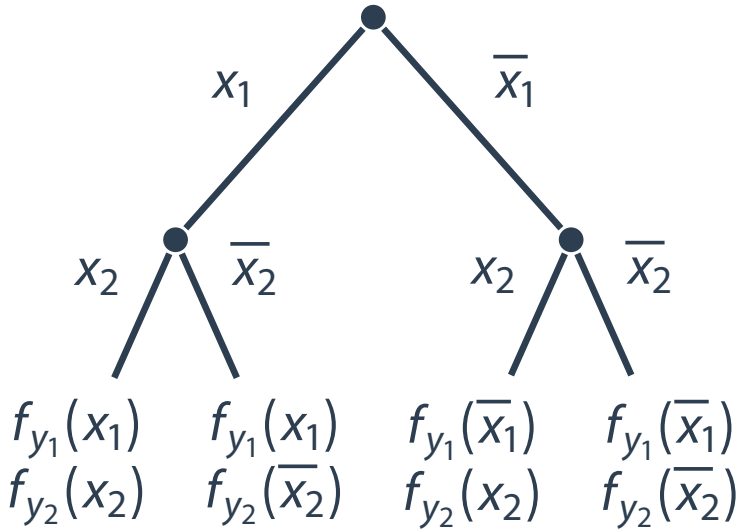
Models of DQBF

A *candidate model* is a composition of Skolem functions

Example ($\forall x_1, x_2.\, \exists_{\{x_1\}} y_1.\, \exists_{\{x_2\}} y_2.\, \varphi$)



$\Rightarrow$



Definition (Model M)

A model is a candidate model where all paths satisfy φ.

DQBF Solving

Exact methods

- Fröhlich et al.: A DPLL Algorithm for Solving DQBF
- Gitina et al.: non-public expansion-based solver
- Fröhlich et al.: iDQ, instantiation-based DQBF solver

QBF Abstraction

Gitina et al.: Equivalence checking of partial designs using DQBF

Expansion-Based Solving

Idea: **Expand the whole BDT**

$$\forall x_1, x_2. \exists_{\{x_1\}} y_1. \exists_{\{x_2\}} y_2. \varphi$$

Eliminate quantifier x_1:

$$\forall x_2. \exists_{\{x_2\}} y_2. \exists_{\varnothing} y_1, y_1'. \varphi|_{x_1=0} \wedge \varphi'|_{x_1=1}$$

Exponential blow-up

QBF Abstraction

Idea: **Linearize the Quantification**

$$\forall x_1, x_2.\ \exists_{\{x_1\}} y_1.\ \exists_{\{x_2\}} y_2.\ \varphi$$

3 possible linearizations:

$$\forall x_1, x_2.\ \exists y_1, y_2. \qquad \text{weakest}$$

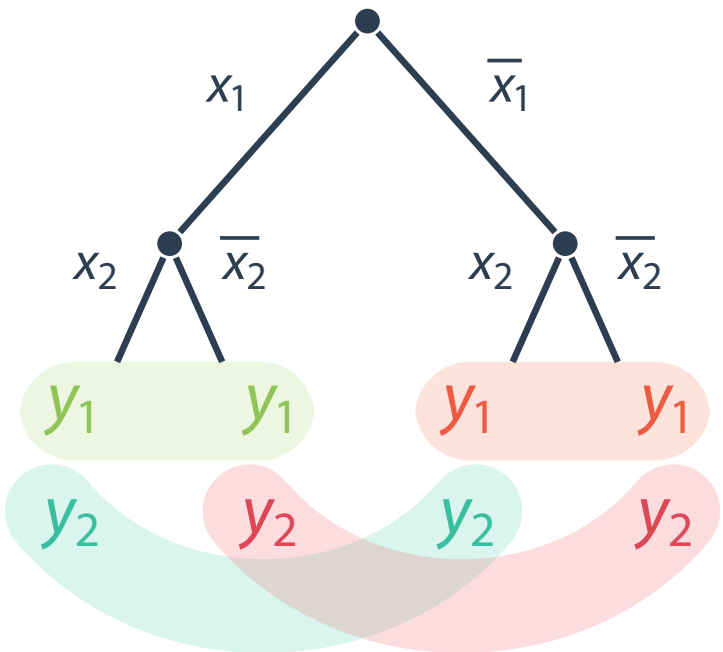
$$\left.\begin{array}{l} \forall x_1.\ \exists y_1.\ \forall x_2.\ \exists y_2. \\ \forall x_2.\ \exists y_2.\ \forall x_1.\ \exists y_1. \end{array}\right\} \qquad \text{strongest}$$

Too coarse: All QBF abstractions of
$\forall x_1, x_2.\ \exists_{\{x_1\}} y_1.\ \exists_{\{x_2\}} y_2.\ (y_1 \vee y_2) \leftrightarrow (x_1 \otimes x_2)$ are satisfiable

Consistency

- Another characterization of Skolem functions
- Property **between** paths

Different assignments of y on two paths P_1 and P_2 are *consistent* if the assignments of H_y on P_1 and P_2 are different

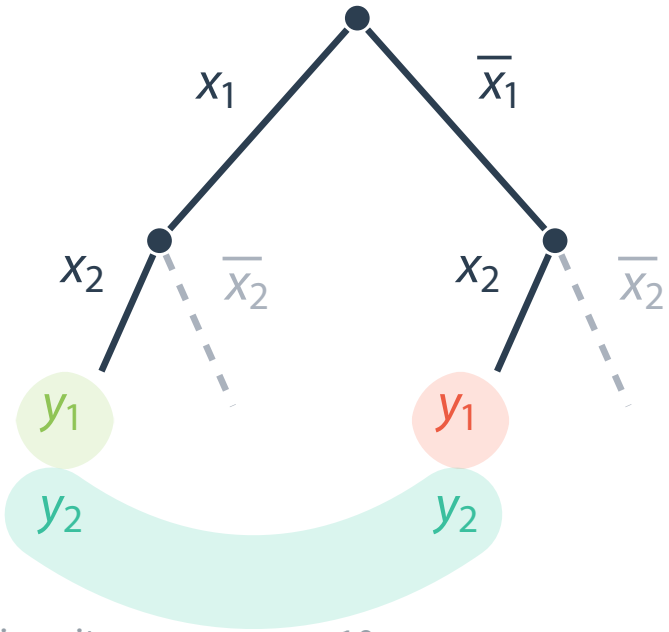


Bounded Unsatisfiability

Unsatisifability can be usually shown without expanding the whole BDT

A set of paths $\mathcal{P}$ can already rule out any satisfying assignment:

If there is no *consistent* satisfying assignment on $\mathcal{P}$, then there is no consistent satisfying assignment for the original formula



Bounded Unsatisfiability

Definition (*k*-bounded unsatisfiable)

For a $k \geq 1$, a DQBF formula Φ is *k-bounded unsatisfiable* if there exists a set of paths $\mathcal{P}$ with $|\mathcal{P}| \leq k$ such that there does not exist a consistent satisfying assignment over $\mathcal{P}$.

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Theorem

A DQBF formula Φ is unsatisfiable iff it is k-bounded unsatisfiable for some $k \geq 1$.

Encoding of Bounded Unsat. in QBF

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bunsat($\Phi, k = 2$) =

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$$\text{bunsat}(\Phi, k = 2) = \exists x_1^1, x_1^2, x_2^1, x_2^2.$$

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$$\bigwedge_{i \in \{1,2\}} ((y_i^1 \leftrightarrow y_i^2) \vee (x_i^1 \not\leftrightarrow x_i^2)) \rightarrow$$
$$\neg\varphi^1 \vee \neg\varphi^2$$

Encoding of Bounded Unsat. in QBF

$$\Phi := \forall x_1, x_2, \ldots x_m . \exists_{H_1} y_1 . \exists_{H_2} y_2 \ldots \exists_{H_n} y_n . \varphi$$

$$\begin{aligned}\text{bunsat}(\Phi, k) := \exists & x_1^1, \ldots, x_m^1, x_1^2, \ldots, x_m^2, \ldots, x_m^k . \\ \forall & y_1^1, \ldots, y_n^1, y_1^2, \ldots, y_n^2, \ldots, y_n^k . \\ & \text{consistent}(\{y_1, \ldots, y_n\}, k) \rightarrow \bigvee_{1 \leq i \leq k} \neg \varphi^k\end{aligned}$$

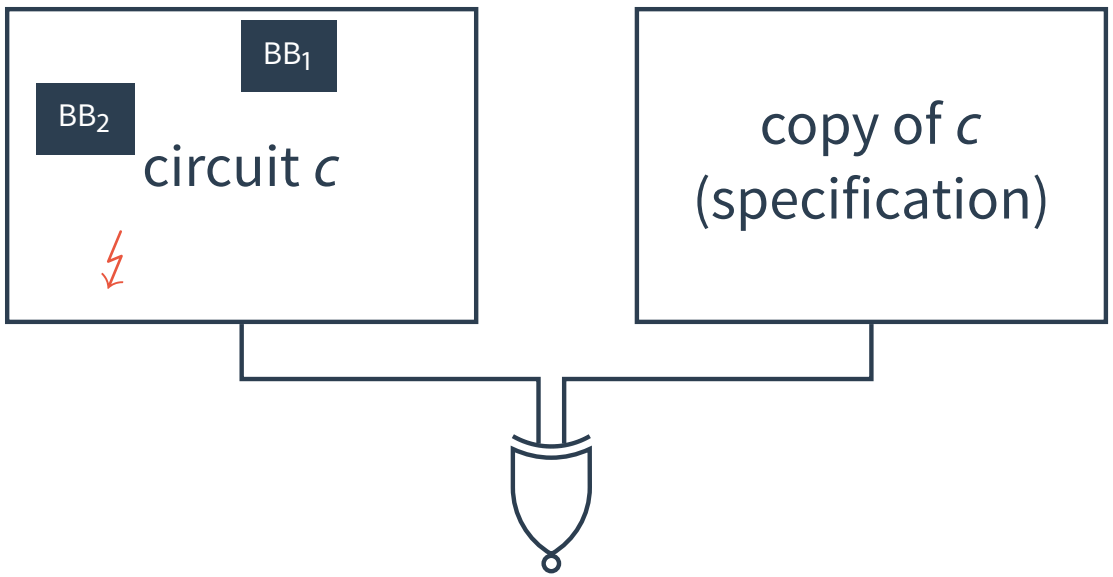
$$\text{consistent}(Y, k) := \bigwedge_{y \in Y} \bigwedge_{(i,j) \in \{1,\ldots,k\}^2} \left((y^i \leftrightarrow y^j) \vee \left(\bigvee_{x \in H_y} x^i \not\leftrightarrow x^j \right) \right)$$

Size of bunsat(Φ, k)

$|X'| = k \cdot |X|$
$|Y'| = k \cdot |Y|$
$|\varphi'| = O(|Y| \cdot k^2 \cdot \max_{y \in Y} |H_y| + k \cdot |\varphi|)$

Experimental Results
PEC Benchmarks

- There were no publicly available benchmarks for DQBF
- Build PEC examples[2]
  - Base: arithmetic circuit
  - One copy as specification
  - Introduce faults and black boxes randomly



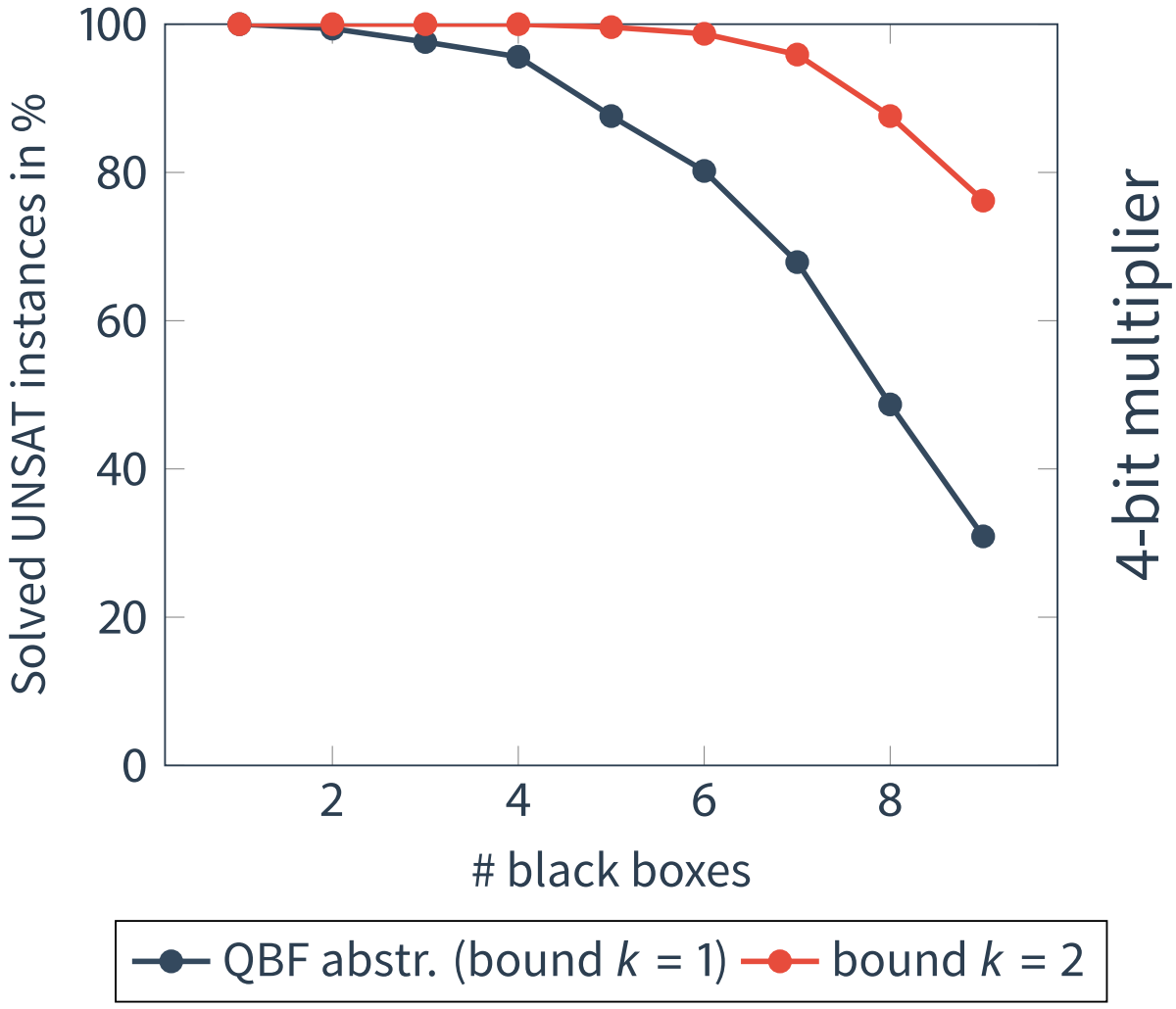
[2] Karina Gitina et al. "Equivalence checking of partial designs using dependency quantified Boolean formulae". In: *ICCD*. IEEE, 2013, pp. 396–403

Experimental Results
Solvers

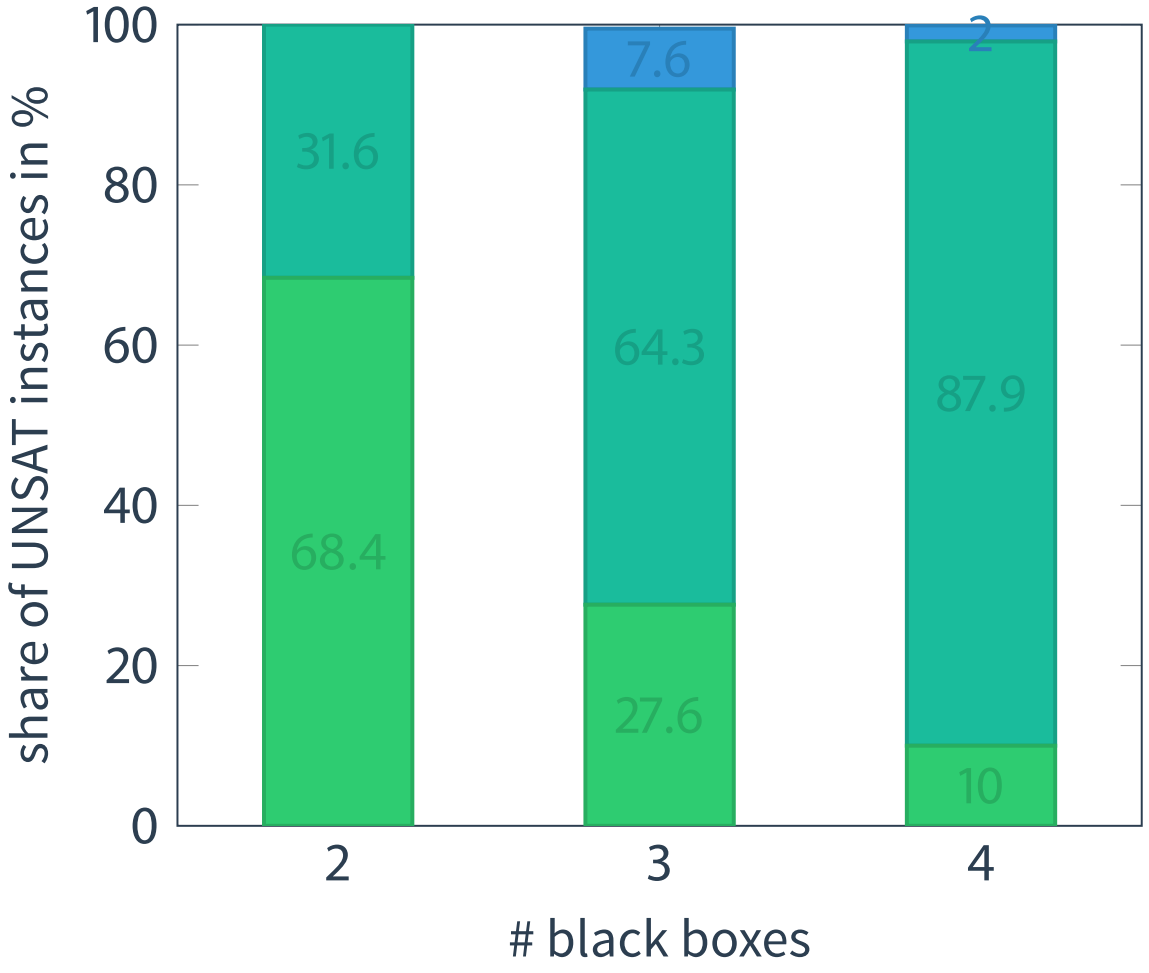
- Bounded Unsatisfiability:
  - Prototype implementing the transformation
  - Optimization: Combine with QBF abstraction
  - QBF queries: Bloqqer 031 and DepQBF 2.0
- Solvers:
  - There were no publicly available DQBF solver
  - ➤ Build an expansion-based solver on top of CUDD

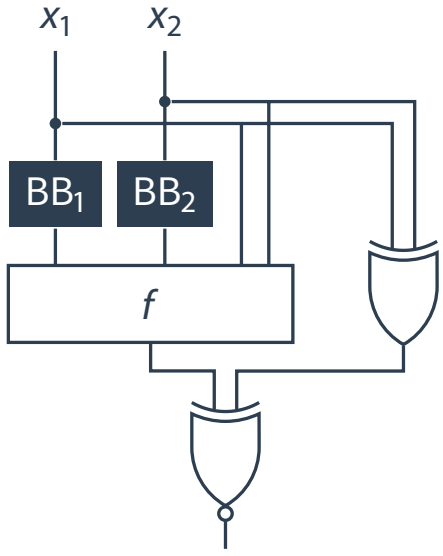
All sources are available at
`http://react.uni-saarland.de/tools/bunsat/`

Number of Solved Instances



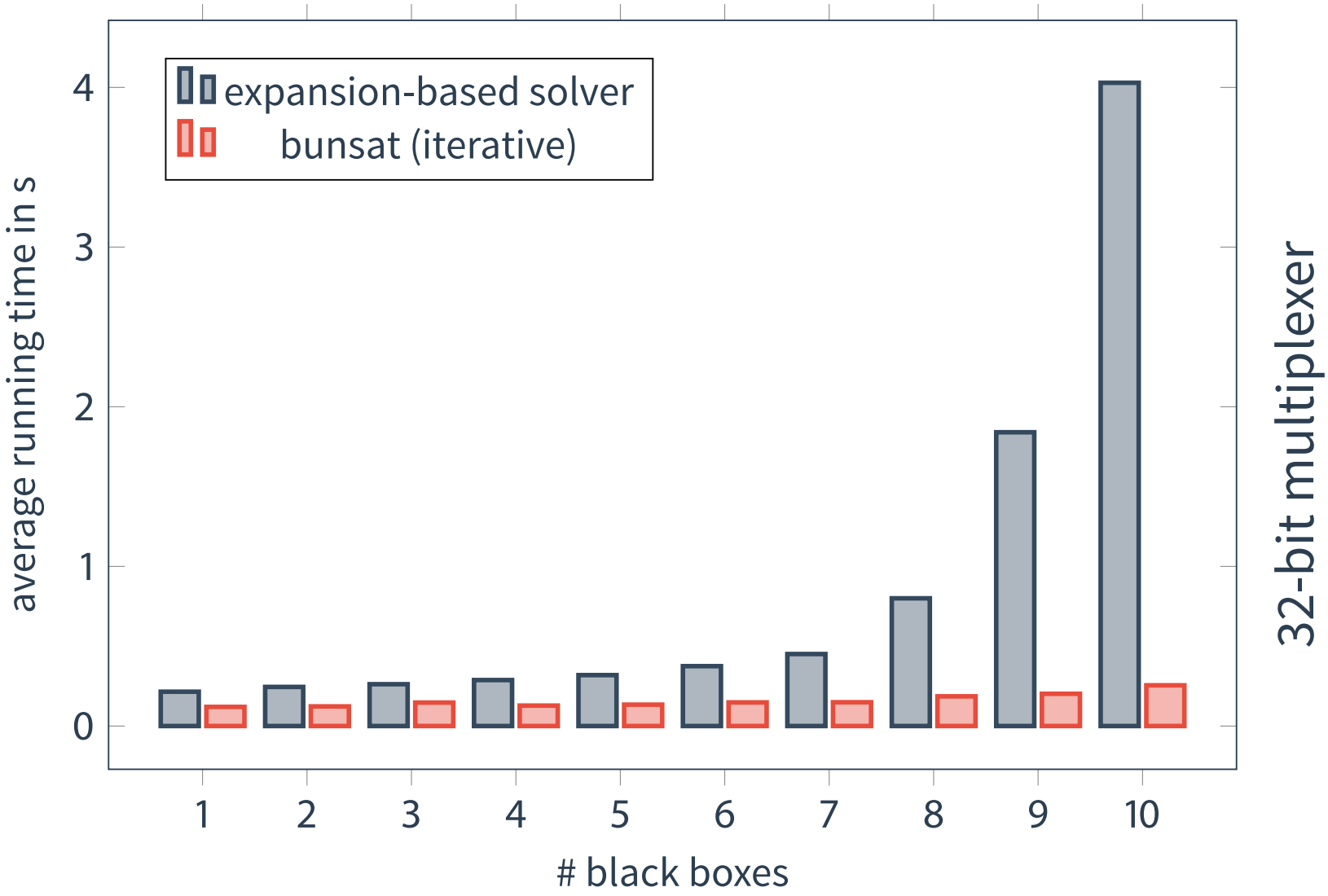
Precision





QBF abstr. (bound $k = 1$) bound $k = 2$ bound $k = 3$

Performance



Conclusion

- Fast DQBF refutation
- Improvement over
  - QBF abstraction (approximation quality)
  - expansion based methods (better scaling with respect to number of black boxes)

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Thank You!