Open-WBO: a Modular MaxSAT Solver

Ruben Martins[⋆], Vasco Manquinho[†], Inês Lynce[†]

[⋆]University of Oxford [†]INESC-ID / IST, University of Lisbon

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What is Boolean Satisfiability?

CNF Formula:

$$\bar{x}_2 \vee \bar{x}_1 \qquad x_2 \vee \bar{x}_3 \qquad x_1$$

$$x_3 \qquad x_2 \vee \bar{x}_1 \qquad \bar{x}_3 \vee x_1$$

- Boolean Satisfiability (SAT):
  - Decide if the formula is satisfiable or unsatisfiable

What is Boolean Satisfiability?

CNF Formula:

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- Formula is unsatisfiable

What is Boolean Satisfiability?

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- Formula is unsatisfiable
- How many clauses can we satisfy?

What is Maximum Satisfiability?

CNF Formula:

$$\bar{x}_2 \vee \bar{x}_1 \qquad x_2 \vee \bar{x}_3 \qquad x_1$$

$$x_3 \qquad x_2 \vee \bar{x}_1 \qquad \bar{x}_3 \vee x_1$$

- Maximum Satisfiability (MaxSAT):
  - Find an assignment that maximizes (minimizes) number of satisfied (unsatisfied) clauses

What is Maximum Satisfiability?

CNF Formula:

$\bar{x}_2 \vee \bar{x}_1$	$x_2 \vee \bar{x}_3$	x_1
x_3	$x_2 \vee \bar{x}_1$	$\bar{x}_3 \vee x_1$

- An optimal solution would be:
  - $\nu = \{x_1 = 1, x_2 = 1, x_3 = 1\}$
- This assignment unsatisfies only 1 clause

MaxSAT Problems

- MaxSAT:
  - All clauses are soft
  - Minimize number of unsatisfied soft clauses
- Partial MaxSAT:
  - Clauses are soft or hard
  - Hard clauses must be satisfied
  - Minimize number of unsatisfied soft clauses
- Weighted Partial MaxSAT:
  - Clauses are soft or hard
  - Weights associated with soft clauses
  - Minimize sum of weights of unsatisfied soft clauses

Why is MaxSAT Important?

- Many real-world applications can be encoded to MaxSAT:
  - Software package upgradeability:
    - Eclipse platform uses MaxSAT for managing the plugins dependencies
  - Error localization in C code
  - Debugging of hardware designs
  - Haplotyping with pedigrees
  - Reasoning over Biological Networks
  - Course timetabling
  - Combinatorial auctions
  - ...
- MaxSAT algorithms are effective for solving real-word problems

Why a modular open source MaxSAT solver?

- Open source MaxSAT solvers are scarce
- Most MaxSAT solvers are tightly coupled with the SAT solver

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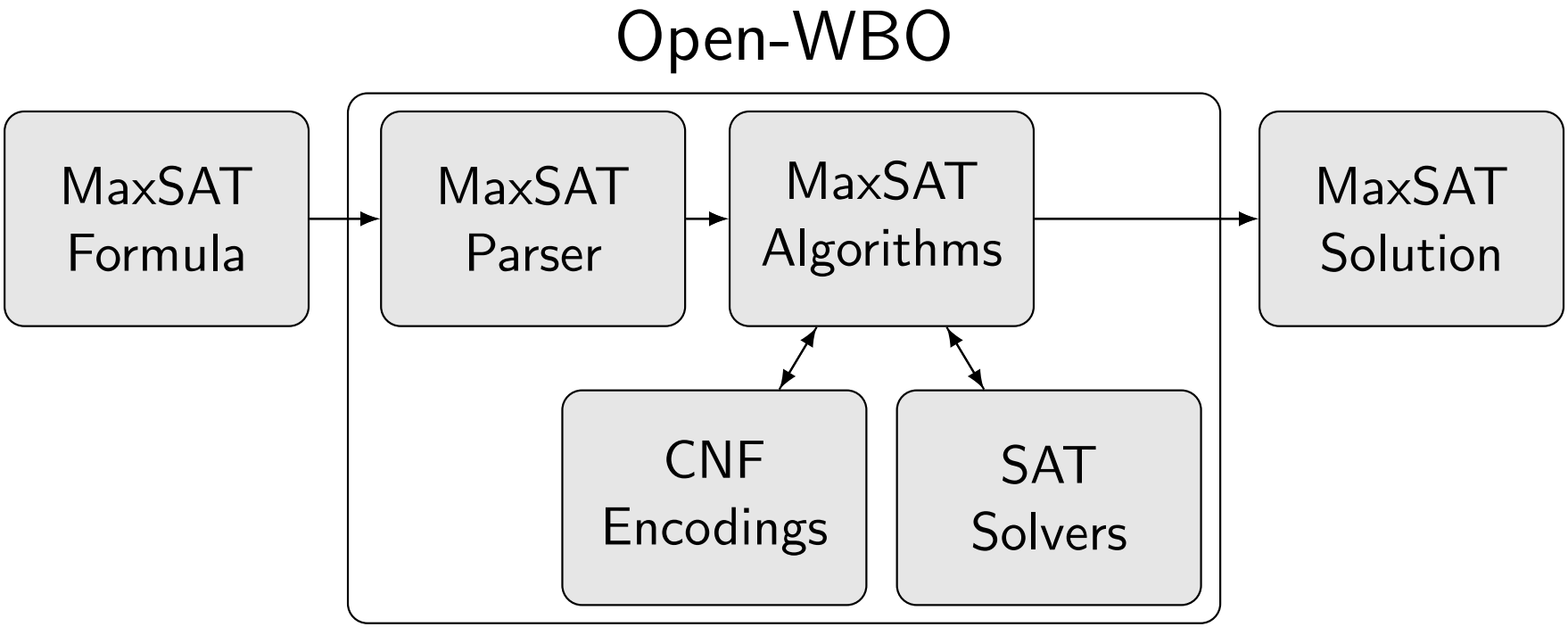
Open-WBO:

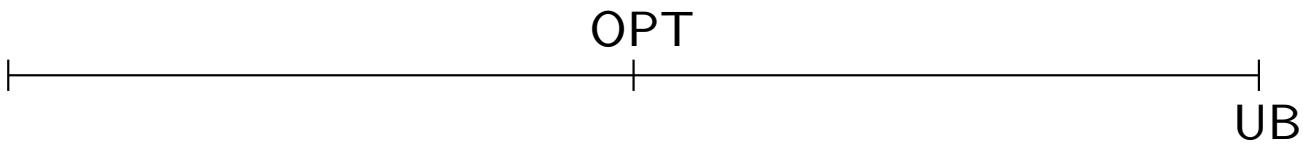
- Open source:
  - Competitive baseline solver
  - Easy to modify and extend
- Modular:
  - Uses the SAT solver as a black box
  - Any MiniSAT-like solver can be easily plugged in
  - Takes advantage of the advances in SAT technology

Open-WBO: architecture



Open-WBO: MaxSAT algorithms

Linear Search algorithms:

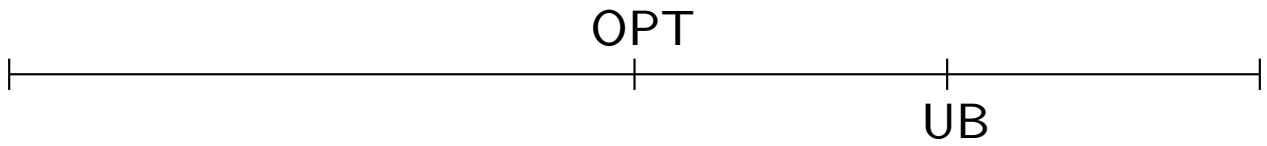


- Optimum solution (OPT):
  - Assignment with **minimum** cost
- Upper Bound (UB) value:
  - Cost **greater than or equal** to OPT

- Linear search algorithms:
  - Refine UB value until OPT is found

Open-WBO: MaxSAT algorithms

Linear Search algorithms:



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Open-WBO: MaxSAT algorithms

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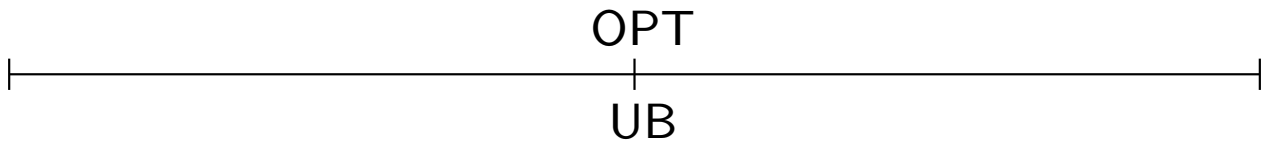

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Open-WBO: linear search algorithms

Partial MaxSAT Formula:

φ_h (Hard): $\quad \bar{x}_2 \vee \bar{x}_1 \quad x_2 \vee \bar{x}_3$

φ_s (Soft): $\quad x_1 \quad x_3 \quad x_2 \vee \bar{x}_1 \quad \bar{x}_3 \vee x_1$

Open-WBO: linear search algorithms

Partial MaxSAT Formula:

φ_h : $\quad \bar{x}_2 \vee \bar{x}_1 \quad x_2 \vee \bar{x}_3$

φ_s : $\quad x_1 \vee r_1 \quad x_3 \vee r_2 \quad x_2 \vee \bar{x}_1 \vee r_3 \quad \bar{x}_3 \vee x_1 \vee r_4$

- Relax all soft clauses
- Relaxation variables:
  - $V_R = \{r_1, r_2, r_3, r_4\}$
  - If a soft clause ω_i is **unsatisfied**, then $r_i = 1$
  - If a soft clause ω_i is **satisfied**, then $r_i = 0$

Open-WBO: linear search algorithms

Partial MaxSAT Formula:

φ_h : $\quad \bar{x}_2 \vee \bar{x}_1 \qquad x_2 \vee \bar{x}_3$

φ_s : $\quad x_1 \vee r_1 \qquad x_3 \vee r_2 \qquad x_2 \vee \bar{x}_1 \vee r_3 \qquad \bar{x}_3 \vee x_1 \vee r_4$

$V_R = \{r_1, r_2, r_3, r_4\}$

- Formula is satisfiable
  - $\nu = \{x_1 = 1, x_2 = 0, x_3 = 0, r_1 = 0, r_2 = 1, r_3 = 1, r_4 = 0\}$
- **Goal:** Minimize the number of relaxation variables assigned to 1

Open-WBO: linear search algorithms

Partial MaxSAT Formula:

φ_h : $\bar{x}_2 \vee \bar{x}_1$ $x_2 \vee \bar{x}_3$

φ_s : $x_1 \vee r_1$ $x_3 \vee r_2$ $x_2 \vee \bar{x}_1 \vee r_3$ $\bar{x}_3 \vee x_1 \vee r_4$

$\mu = 2 \quad V_R = \{r_1, r_2, r_3, r_4\}$

- r_2 and r_3 were assigned truth value 1:
  - Current solution unsatisfies 2 soft clauses
- Can less than 2 soft clauses be unsatisfied?

Open-WBO: linear search algorithms

Partial MaxSAT Formula:

φ_h : $\quad \bar{x}_2 \vee \bar{x}_1 \quad x_2 \vee \bar{x}_3 \quad \text{CNF}(\sum_{r_i \in V_R} r_i \leq 1)$

φ_s : $\quad x_1 \vee r_1 \quad x_3 \vee r_2 \quad x_2 \vee \bar{x}_1 \vee r_3 \quad \bar{x}_3 \vee x_1 \vee r_4$

$\mu = 2 \quad V_R = \{r_1, r_2, r_3, r_4\}$

- Add cardinality constraint to refine UB value:
  - $\text{CNF}(r_1 + r_2 + r_3 + r_4 \leq 1)$

Open-WBO: linear search algorithms

Partial MaxSAT Formula:

φ_h : $\bar{x}_2 \vee \bar{x}_1$ $x_2 \vee \bar{x}_3$ $\text{CNF}(\sum_{r_i \in V_R} r_i \leq 1)$

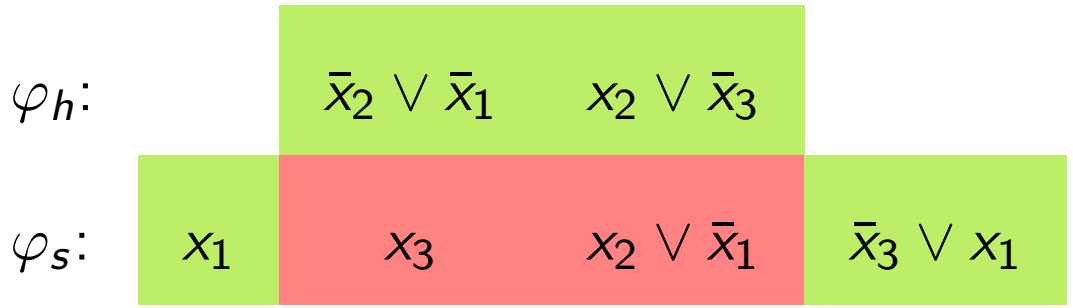
φ_s : $x_1 \vee r_1$ $x_3 \vee r_2$ $x_2 \vee \bar{x}_1 \vee r_3$ $\bar{x}_3 \vee x_1 \vee r_4$

$\mu = 2$ $V_R = \{r_1, r_2, r_3, r_4\}$

- Formula is unsatisfiable:
  - There are no solutions that unsatisfy 1 or less soft clauses

Open-WBO: linear search algorithms

Partial MaxSAT Formula:

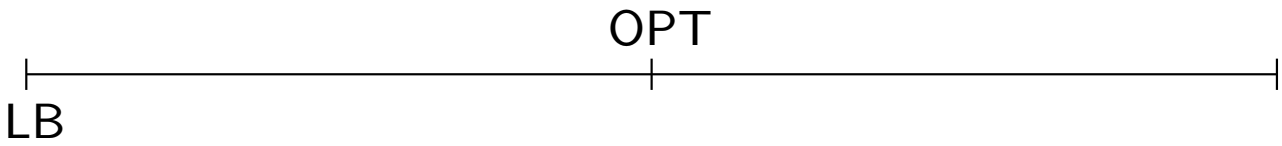


$\mu = 2 \qquad V_R = \{r_1, r_2, r_3, r_4\}$

- **Optimal solution**:
  - $\nu = \{x_1 = 1, x_2 = 0, x_3 = 0\}$

Open-WBO: MaxSAT algorithms

Unsatisfiability-based algorithms:

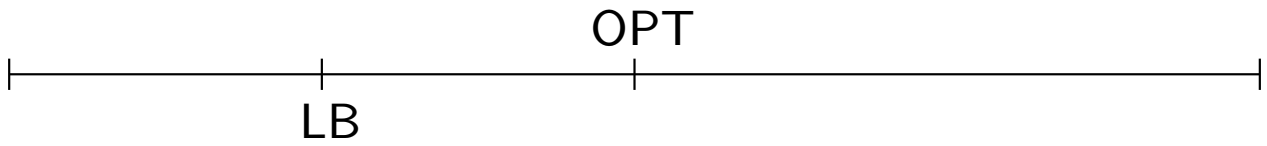


- Lower Bound (LB) value:
  - Cost **smaller than or equal** to OPT

- Unsatisfiability-based algorithms:
  - Use unsatisfiable subformulas to refine LB value until OPT is found

Open-WBO: MaxSAT algorithms

Unsatisfiability-based algorithms:



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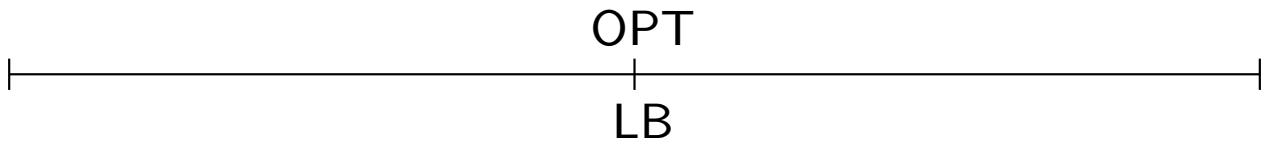
Open-WBO: MaxSAT algorithms

Unsatisfiability-based algorithms:



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Open-WBO: CNF encodings

Type	Encoding	Variables	Clauses
$x_1 + \ldots + x_n \leq 1$	Ladder	$\mathcal{O}(n)$	$\mathcal{O}(n)$
$x_1 + \ldots + x_n \leq k$	Card. Networks	$\mathcal{O}(nlog^2k)$	$\mathcal{O}(nlog^2k)$
$a_1x_1 + \ldots + a_nx_n \leq k$	Weighted Sequential	$\mathcal{O}(nk)$	$\mathcal{O}(nk)$

- Unsatisfiability-based algorithm:
  - Ladder encoding
- Linear search algorithm:
  - MaxSAT, Partial MaxSAT: Card. Networks
  - Weighted Partial MaxSAT: Weighted Sequential

Open-WBO: SAT solvers

- Open-WBO can use any MiniSAT-like SAT solver
- The following solvers can be chosen when compiling Open-WBO:
  - MiniSAT2.0
  - MiniSAT2.2
  - Glucose2.3
  - Glucose3.0
  - GluH
  - ZENN
  - SINN
  - GlueMiniSAT
  - glue_bit
  - GlucoRed

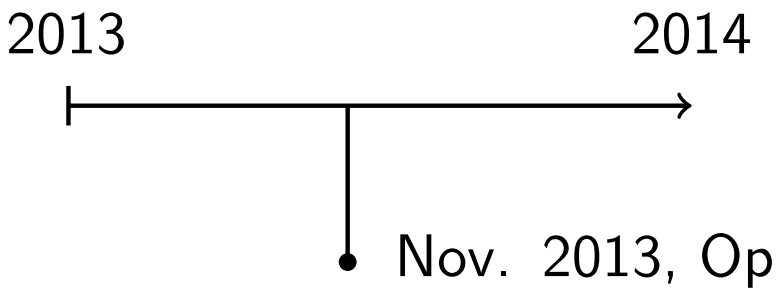
Open-WBO: Demo

Tool Demo!

Impact of different SAT solvers

- Heuristics that are good for SAT instances may not be good for MaxSAT instances
- Assumptions:
  - Fixed decision literals that are chosen first by the SAT solver
  - Assumptions are used for extracting unsatisfiable subformulas
- Assumptions affect the heuristics of SAT solvers [Audemard et al. SAT'13]:
  - Only Glucose3.0 has optimizations when using assumptions
- When using assumptions:
  - Most MiniSAT-like solvers have **similar performance** to MiniSAT2.2
- When not using assumptions:
  - Most MiniSAT-like solvers have **better performance** than MiniSAT2.2

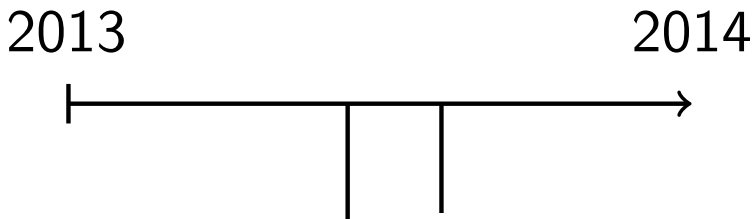
Timeline of Open-WBO



Nov. 2013, Open-WBO v1.0:

- Algorithms: WBO
- Encodings: Ladder (at-most-one)
- Solvers: Glucose3.0, MiniSAT2.2

Timeline of Open-WBO

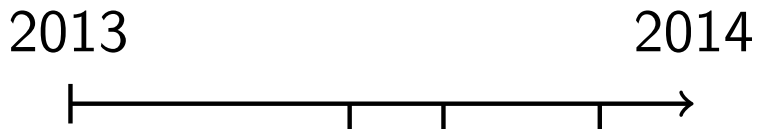


Nov. 2013, Open-WBO v1.0

Feb. 2014, Open-WBO v1.1:

- Algorithms: Linear Search Sat-Unsat
- Encodings: Cardinality Networks (at-most-k), Weighted Sequential (PB constraints)
- Solvers: MiniSAT2.0, Glucose2.3, ZENN, SINN, GlueMiniSAT, GluH, glue_bit, GlucoRed

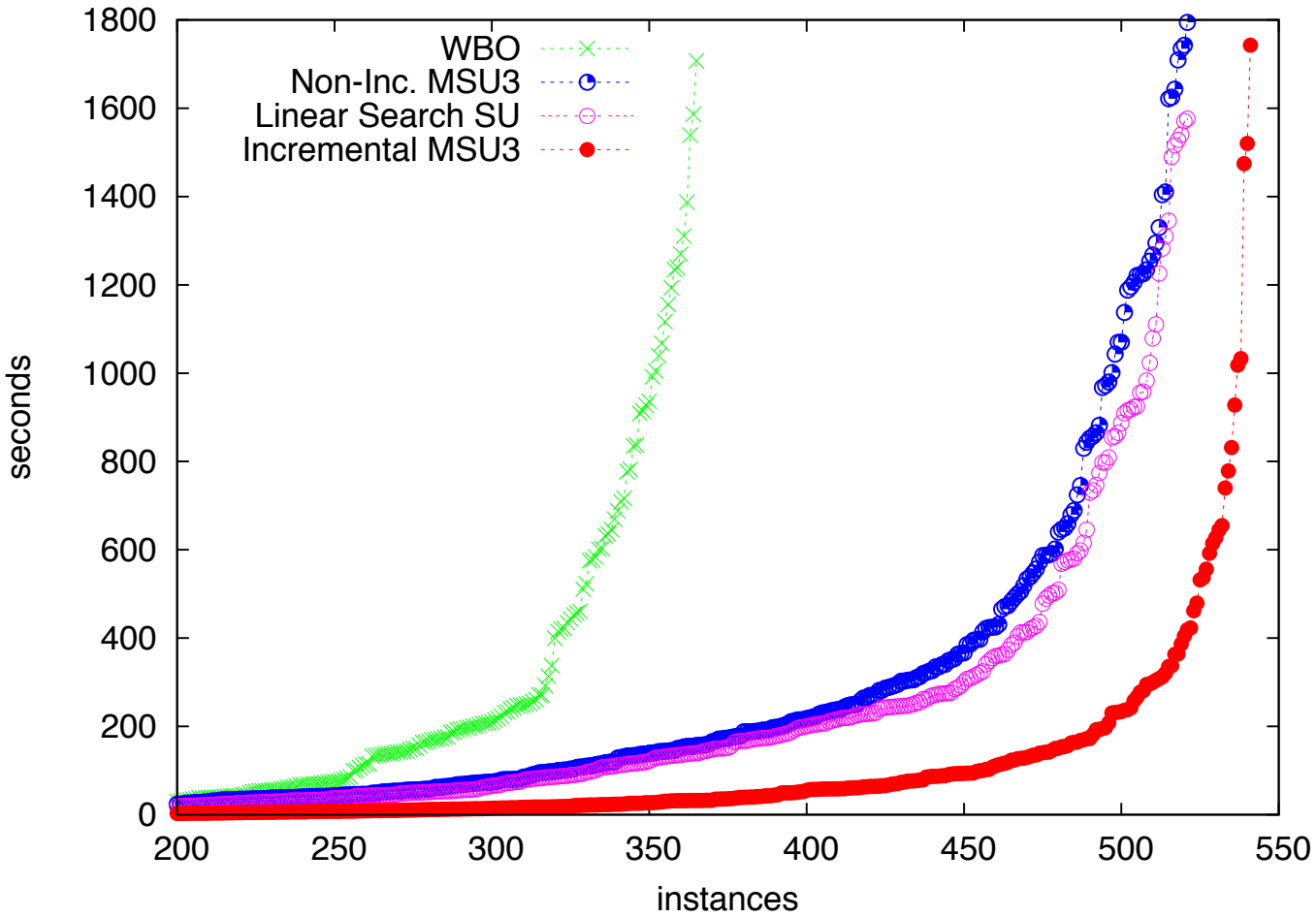
Timeline of Open-WBO



- Nov. 2013, Open-WBO v1.0
- Feb. 2014, Open-WBO v1.1
- **New**: Jul. 2014, Open-WBO v1.2:
  - Algorithms: Linear Search Unsat-Sat, MSU3,
    Novel: Incremental Unsat-based Algorithms
  - Encodings: Totalizer, Modulo Totalizer,
    Novel: Incremental Totalizer (at-most-k)

Incremental Cardinality Constraints for MaxSAT [Martins et al., CP'14]

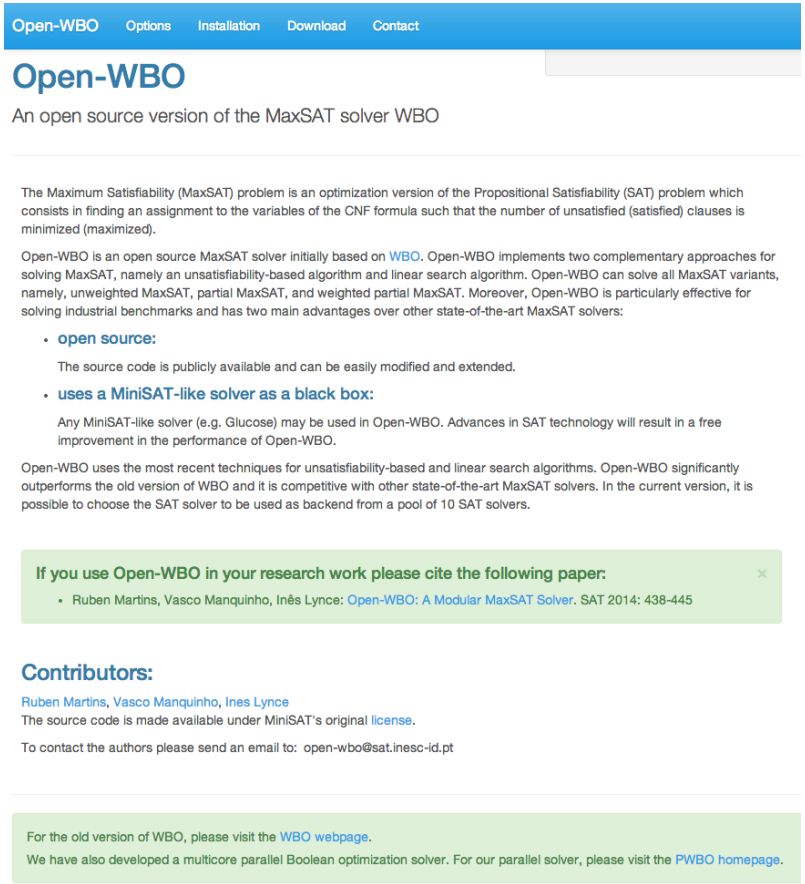
- Incremental MaxSAT algorithms on partial MaxSAT instances:



Open-WBO and state-of-the-art MaxSAT solvers

- Open-WBO is competitive with state-of-the-art MaxSAT solvers
- Open-WBO participated in the MaxSAT Evaluation 2014:
  - All versions were submitted with Glucose 3.0
  - Open-WBO:
    - Alg.: WBO, Enc:. Ladder
  - Open-WBO-SU:
    - Alg.: Linear Search Sat-Unsat,
      Enc.: Modulo Totalizer & Weighted Sequential
  - Open-WBO-Inc:
    - Alg.: Incremental MSU3, Enc.: Incremental Totalizer
- Results will be out on Thursday, July 17th!

Try out Open-WBO!



webpage:
http://sat.inesc-id.pt/open-wbo/

contact:
open-wbo@sat.inesc-id.pt

Comments and suggestions are welcome and will help us in improving Open-WBO!