

Open-WBO: a Modular MaxSAT Solver

Ruben Martins^{*}, Vasco Manquinho[†], Inês Lynce[†]

^{*}University of Oxford

[†]INESC-ID / IST, University of Lisbon



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What is Boolean Satisfiability?

CNF Formula:

$$\begin{array}{ccc} \bar{x}_2 \vee \bar{x}_1 & x_2 \vee \bar{x}_3 & x_1 \\ x_3 & x_2 \vee \bar{x}_1 & \bar{x}_3 \vee x_1 \end{array}$$

- Boolean Satisfiability (SAT):
 - Decide if the formula is satisfiable or unsatisfiable

What is Boolean Satisfiability?

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- Formula is unsatisfiable

What is Boolean Satisfiability?

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- Formula is unsatisfiable
- How many clauses can we satisfy?

What is Maximum Satisfiability?

CNF Formula:

$$\begin{array}{ccc} \bar{x}_2 \vee \bar{x}_1 & x_2 \vee \bar{x}_3 & x_1 \\ x_3 & x_2 \vee \bar{x}_1 & \bar{x}_3 \vee x_1 \end{array}$$

- Maximum Satisfiability (MaxSAT):
 - Find an assignment that maximizes (minimizes) number of satisfied (unsatisfied) clauses

What is Maximum Satisfiability?

CNF Formula:

$\bar{x}_2 \vee \bar{x}_1$	$x_2 \vee \bar{x}_3$	x_1
x_3	$x_2 \vee \bar{x}_1$	$\bar{x}_3 \vee x_1$

- An optimal solution would be:
 - $\nu = \{x_1 = 1, x_2 = 1, x_3 = 1\}$
- This assignment unsatisfies only 1 clause

MaxSAT Problems

- MaxSAT:
 - All clauses are soft
 - Minimize number of unsatisfied soft clauses
- Partial MaxSAT:
 - Clauses are soft or hard
 - Hard clauses must be satisfied
 - Minimize number of unsatisfied soft clauses
- Weighted Partial MaxSAT:
 - Clauses are soft or hard
 - Weights associated with soft clauses
 - Minimize sum of weights of unsatisfied soft clauses

Why is MaxSAT Important?

- Many real-world applications can be encoded to MaxSAT:
 - Software package upgradeability:
 - Eclipse platform uses MaxSAT for managing the plugins dependencies
 - Error localization in C code
 - Debugging of hardware designs
 - Haplotyping with pedigrees
 - Reasoning over Biological Networks
 - Course timetabling
 - Combinatorial auctions
 - ...
- MaxSAT algorithms are effective for solving real-world problems

Why a modular open source MaxSAT solver?

- Open source MaxSAT solvers are scarce
- Most MaxSAT solvers are tightly coupled with the SAT solver

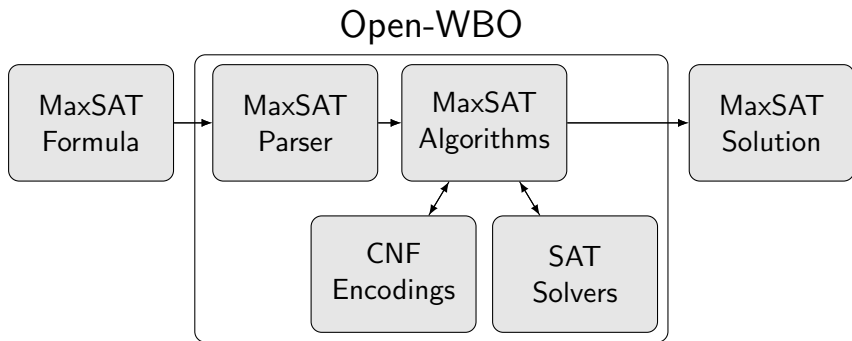
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Open-WBO:

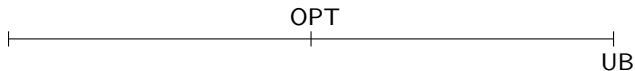
- Open source:
 - Competitive baseline solver
 - Easy to modify and extend
- Modular:
 - Uses the SAT solver as a black box
 - Any MiniSAT-like solver can be easily plugged in
 - Takes advantage of the advances in SAT technology

Open-WBO: architecture



Open-WBO: MaxSAT algorithms

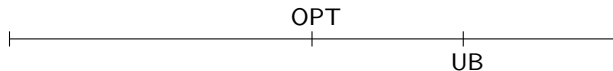
Linear Search algorithms:



- Optimum solution (OPT):
 - Assignment with **minimum** cost
- Upper Bound (UB) value:
 - Cost **greater than or equal** to OPT
- Linear search algorithms:
 - Refine UB value until OPT is found

Open-WBO: MaxSAT algorithms

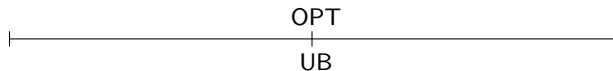
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Open-WBO: MaxSAT algorithms

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Open-WBO: linear search algorithms

Partial MaxSAT Formula:

$$\varphi_h \text{ (Hard):} \quad \bar{x}_2 \vee \bar{x}_1 \quad x_2 \vee \bar{x}_3$$

$$\varphi_s \text{ (Soft):} \quad x_1 \quad x_3 \quad x_2 \vee \bar{x}_1 \quad \bar{x}_3 \vee x_1$$

Open-WBO: linear search algorithms

Partial MaxSAT Formula:

$$\varphi_h : \quad \bar{x}_2 \vee \bar{x}_1 \quad x_2 \vee \bar{x}_3$$

$$\varphi_s : \quad x_1 \vee r_1 \quad x_3 \vee r_2 \quad x_2 \vee \bar{x}_1 \vee r_3 \quad \bar{x}_3 \vee x_1 \vee r_4$$

- Relax all soft clauses
- Relaxation variables:
 - $V_R = \{r_1, r_2, r_3, r_4\}$
 - If a soft clause ω_i is **unsatisfied**, then $r_i = 1$
 - If a soft clause ω_i is **satisfied**, then $r_i = 0$

Open-WBO: linear search algorithms

Partial MaxSAT Formula:

$\varphi_h :$

$$\bar{x}_2 \vee \bar{x}_1$$

$$x_2 \vee \bar{x}_3$$

$\varphi_s :$

$$x_1 \vee r_1$$

$$x_3 \vee r_2$$

$$x_2 \vee \bar{x}_1 \vee r_3$$

$$\bar{x}_3 \vee x_1 \vee r_4$$

$$V_R = \{r_1, r_2, r_3, r_4\}$$

- Formula is satisfiable
 - $\nu = \{x_1 = 1, x_2 = 0, x_3 = 0, r_1 = 0, r_2 = 1, r_3 = 1, r_4 = 0\}$
- **Goal:** Minimize the number of relaxation variables assigned to 1

Open-WBO: linear search algorithms

Partial MaxSAT Formula:

$\varphi_h :$

$\bar{x}_2 \vee \bar{x}_1$

$x_2 \vee \bar{x}_3$

$\varphi_s :$

$x_1 \vee r_1$

$x_3 \vee r_2$

$x_2 \vee \bar{x}_1 \vee r_3$

$\bar{x}_3 \vee x_1 \vee r_4$

$$\mu = 2 \quad V_R = \{r_1, r_2, r_3, r_4\}$$

- r_2 and r_3 were assigned truth value 1:
 - Current solution unsatisfies 2 soft clauses
- Can less than 2 soft clauses be unsatisfied?

Open-WBO: linear search algorithms

Partial MaxSAT Formula:

$$\varphi_h : \quad \bar{x}_2 \vee \bar{x}_1 \quad x_2 \vee \bar{x}_3 \quad \text{CNF}(\sum_{r_i \in V_R} r_i \leq 1)$$

$$\varphi_s : \quad x_1 \vee r_1 \quad x_3 \vee r_2 \quad x_2 \vee \bar{x}_1 \vee r_3 \quad \bar{x}_3 \vee x_1 \vee r_4$$

$$\mu = 2 \quad V_R = \{r_1, r_2, r_3, r_4\}$$

- Add cardinality constraint to refine UB value:
 - $\text{CNF}(r_1 + r_2 + r_3 + r_4 \leq 1)$

Open-WBO: linear search algorithms

Partial MaxSAT Formula:

$$\varphi_h : \quad \bar{x}_2 \vee \bar{x}_1 \quad x_2 \vee \bar{x}_3 \quad \text{CNF}(\sum_{r_i \in V_R} r_i \leq 1)$$

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$$\mu = 2 \quad V_R = \{r_1, r_2, r_3, r_4\}$$

- Formula is unsatisfiable:
 - There are no solutions that unsatisfy 1 or less soft clauses

Open-WBO: linear search algorithms

Partial MaxSAT Formula:

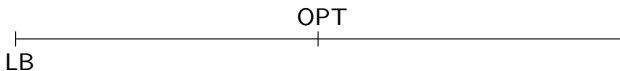
φ_h :		$\bar{x}_2 \vee \bar{x}_1$	$x_2 \vee \bar{x}_3$	
φ_s :	x_1	x_3	$x_2 \vee \bar{x}_1$	$\bar{x}_3 \vee x_1$

$$\mu = 2 \quad V_R = \{r_1, r_2, r_3, r_4\}$$

- **Optimal solution:**
 - $\nu = \{x_1 = 1, x_2 = 0, x_3 = 0\}$

Open-WBO: MaxSAT algorithms

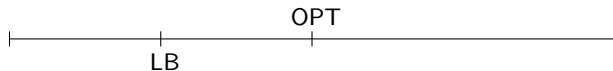
Unsatisfiability-based algorithms:



- Lower Bound (LB) value:
 - Cost **smaller than or equal** to OPT
- Unsatisfiability-based algorithms:
 - Use unsatisfiable subformulas to refine LB value until OPT is found

Open-WBO: MaxSAT algorithms

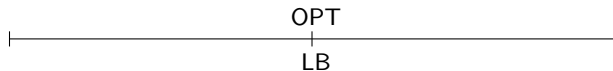
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Open-WBO: MaxSAT algorithms

Unsatisfiability-based algorithms:



- Lower Bound (LB) value:
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Open-WBO: CNF encodings

Type	Encoding	Variables	Clauses
$x_1 + \dots + x_n \leq 1$	Ladder	$\mathcal{O}(n)$	$\mathcal{O}(n)$
$x_1 + \dots + x_n \leq k$	Card. Networks	$\mathcal{O}(n \log^2 k)$	$\mathcal{O}(n \log^2 k)$
$a_1 x_1 + \dots + a_n x_n \leq k$	Weighted Sequential	$\mathcal{O}(nk)$	$\mathcal{O}(nk)$

- Unsatisfiability-based algorithm:
 - Ladder encoding
- Linear search algorithm:
 - MaxSAT, Partial MaxSAT: Card. Networks
 - Weighted Partial MaxSAT: Weighted Sequential

Open-WBO: SAT solvers

- Open-WBO can use any MiniSAT-like SAT solver
- The following solvers can be chosen when compiling Open-WBO:
 - MiniSAT2.0
 - MiniSAT2.2
 - Glucose2.3
 - Glucose3.0
 - GluH
 - ZENN
 - SINN
 - GlueMiniSAT
 - glue_bit
 - GlucoRed

Tool Demo!

Impact of different SAT solvers

- Heuristics that are good for SAT instances may not be good for MaxSAT instances
- Assumptions:
 - Fixed decision literals that are chosen first by the SAT solver
 - Assumptions are used for extracting unsatisfiable subformulas
- Assumptions affect the heuristics of SAT solvers [Audemard et al. SAT'13]:
 - Only Glucose3.0 has optimizations when using assumptions
- When using assumptions:
 - Most MiniSAT-like solvers have **similar performance** to MiniSAT2.2
- When not using assumptions:
 - Most MiniSAT-like solvers have **better performance** than MiniSAT2.2

Timeline of Open-WBO

2013

2014

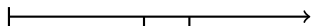


- Nov. 2013, Open-WBO v1.0:
 - Algorithms: WBO
 - Encodings: Ladder (at-most-one)
 - Solvers: Glucose3.0, MiniSAT2.2

Timeline of Open-WBO

2013

2014

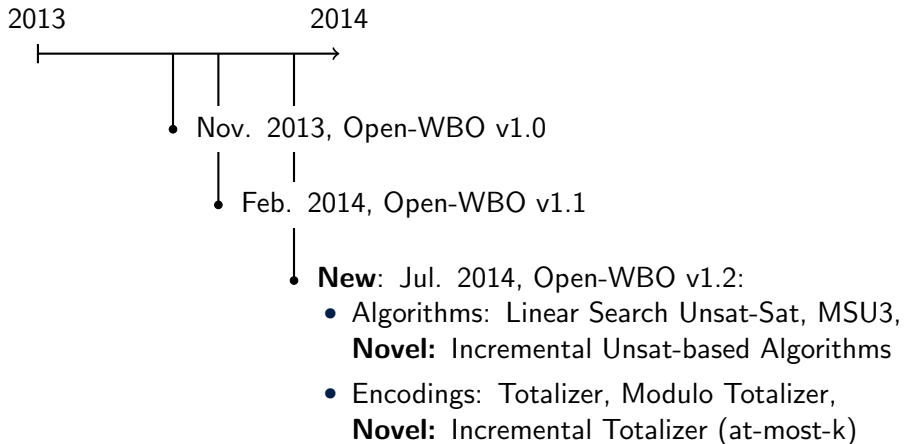


• Nov. 2013, Open-WBO v1.0

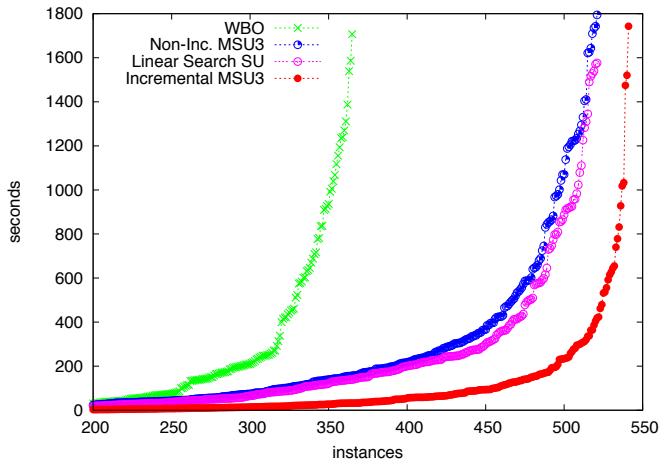
• Feb. 2014, Open-WBO v1.1:

- Algorithms: Linear Search Sat-Unsat
- Encodings: Cardinality Networks (at-most-k), Weighted Sequential (PB constraints)
- Solvers: MiniSAT2.0, Glucose2.3, ZENN, SINN, GlueMiniSAT, GluH, glue_bit, GlucoRed

Timeline of Open-WBO



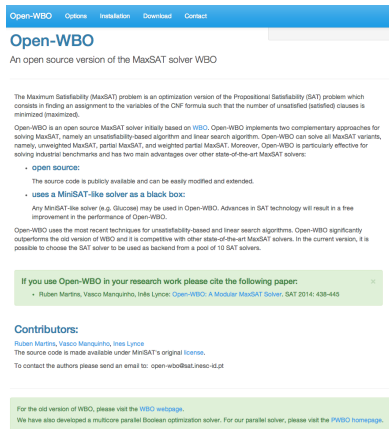
- Incremental MaxSAT algorithms on partial MaxSAT instances:



Open-WBO and state-of-the-art MaxSAT solvers

- Open-WBO is competitive with state-of-the-art MaxSAT solvers
- Open-WBO participated in the MaxSAT Evaluation 2014:
 - All versions were submitted with Glucose 3.0
 - Open-WBO:
 - Alg.: WBO, Enc.: Ladder
 - Open-WBO-SU:
 - Alg.: Linear Search Sat-Unsat,
Enc.: Modulo Totalizer & Weighted Sequential
 - Open-WBO-Inc:
 - Alg.: Incremental MSU3, Enc.: Incremental Totalizer
- Results will be out on Thursday, July 17th!

Try out Open-WBO!



The screenshot shows the Open-WBO website homepage. At the top, there is a navigation bar with links for 'Open-WBO', 'Options', 'Installation', 'Download', and 'Contact'. Below the navigation bar, the title 'Open-WBO' is displayed in a large, bold font, followed by the subtitle 'An open source version of the MaxSAT solver WBO'. The main content area contains a paragraph describing the Maximum Satisfiability (MaxSAT) problem, followed by a paragraph explaining that Open-WBO is an open source MaxSAT solver based on WBO, implementing two complementary approaches for solving MaxSAT. Below this, there are two bullet points: 'open source' and 'uses a MiniSAT-like solver as a black box'. A paragraph follows, mentioning that any MiniSAT-like solver (e.g., Glucose) may be used in Open-WBO. Another paragraph states that Open-WBO uses the most recent techniques for unsatisfiability-based and linear search algorithms. At the bottom of the main content area, there is a green box with the text 'If you use Open-WBO in your research work please cite the following paper:' followed by a citation. Below this, there is a 'Contributors:' section with the names 'Ruben Martins, Vasco Manquinho, Ines Lynce' and a note about the source code being made available under MiniSAT's original license. At the very bottom, there is another green box with text about the old version of WBO and a parallel Boolean optimization solver.

Open-WBO Options Installation Download Contact

Open-WBO

An open source version of the MaxSAT solver WBO

The Maximum Satisfiability (MaxSAT) problem is an optimization version of the Propositional Satisfiability (SAT) problem which consists in finding an assignment to the variables of the CNF formula such that the number of unsatisfied (satisfied) clauses is minimized (maximized).

Open-WBO is an open source MaxSAT solver initially based on WBO. Open-WBO implements two complementary approaches for solving MaxSAT, namely an unsatisfiability-based algorithm and linear search algorithm. Open-WBO can solve all MaxSAT variants, namely, unweighted MaxSAT, partial MaxSAT, and weighted partial MaxSAT. Moreover, Open-WBO is particularly effective for solving industrial benchmarks and has two main advantages over other state-of-the-art MaxSAT solvers:

- **open source:**
The source code is publicly available and can be easily modified and extended.
- **uses a MiniSAT-like solver as a black box:**
Any MiniSAT-like solver (e.g. Glucose) may be used in Open-WBO. Advances in SAT technology will result in a free improvement in the performance of Open-WBO.

Open-WBO uses the most recent techniques for unsatisfiability-based and linear search algorithms. Open-WBO significantly outperforms the old version of WBO and it is competitive with other state-of-the-art MaxSAT solvers. In the current version, it is possible to choose the SAT solver to be used as backend from a pool of 10 SAT solvers.

If you use Open-WBO in your research work please cite the following paper:

- Ruben Martins, Vasco Manquinho, Ines Lynce: [Open-WBO: A Modular MaxSAT Solver](#). SAT 2014: 438-445

Contributors:

Ruben Martins, Vasco Manquinho, Ines Lynce
The source code is made available under MiniSAT's original license.
To contact the authors please send an email to: open-wbo@sat.inesc-id.pt

For the old version of WBO, please visit the [WBO webpage](#).
We have also developed a multicore parallel Boolean optimization solver. For our parallel solver, please visit the [PWBO homepage](#).

webpage:

<http://sat.inesc-id.pt/open-wbo/>

contact:

open-wbo@sat.inesc-id.pt

Comments and suggestions are welcome and will help us in improving Open-WBO!