Exercises for the Lecture Practical SAT Solving | Summer Semester 2019 | Assignment 4

Institute for Theoretical Informatics, Karlsruhe Institute of Technology (KIT) 27.6.2019

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Exercise 1: (Variable Elimination) [3 Points]

In Lecture 9, we presented a variable elimination procedure based on gates, where, for a CNF-representation of a gate G in a formula F with $F = G \dot\cup R$, we replaced $S = (G_x \cup R_x) \cup (G_{\overline{x}} \cup R_{\overline{x}})$ by $S' = (G_x \otimes R_{\overline{x}}) \cup (R_x \otimes G_{\overline{x}})$, dropping both $R_x \otimes R_{\overline{x}}$ and $G_x \otimes G_{\overline{x}}$ from S'. Show that the clauses in $R_x \otimes R_{\overline{x}}$ can be derived by resolution from S', and thus do not have to be included in S'. You may assume that G is a binary AND-gate.

Exercise 2: (Variable Elimination) [4 Points]

Given

$$S = \{\underbrace{\{\neg x, \neg y, a\}, \{x, \neg a\}, \{y, \neg a\}}_{G_1}, \underbrace{\{\neg a, r\}, \{\neg z, r\}, \{a, z, \neg r\}}_{G_2}, \{a, z, r\}, \{\neg a, \neg r\}\} \ .$$

Apply elimination by clause distribution for gates (as presented in the lecture) to S.

1. Eliminate gate G_1 (variable a) first, then gate G_2 (variable r), if possible.
2. Eliminate gate G_2 (variable r) first, then gate G_1 (variable a), if possible.

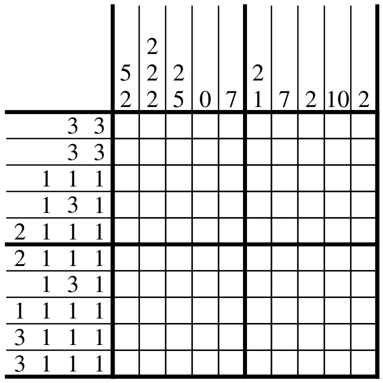
Give the clause sets after each elimination step.

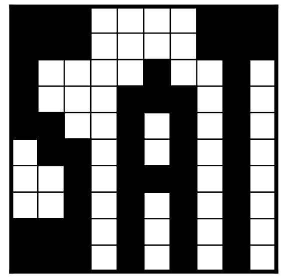
Exercise 3: (Blocked Clauses) [3+3+3 Points]

Blocked clause elimination (BCE) is the process of iteratively removing blocked clauses from a CNF formula until the formula does not contain any more blocked clauses. If BCE reduces a CNF formula F to the empty formula (has no clauses) then F is called a blocked set. Prove the following statements regarding blocked sets (3 points for each)

1. Any CNF F can be split into two blocked sets S and L, i.e., $F = S \cup L$, and both S and L are blocked sets. Design a linear algorithm (in the size of F) that produces L and S from F.
2. Blocked sets are not closed unter resolution, i.e., if F is a blocked set then $F \cup C_1 \otimes C_2$, where $C_1, C_2 \in F$ may not be a blocked set anymore.
3. Blocked sets are not closed unter partially assigning variables, i.e., if F is a blocked set then $F_{x=v}$ may not be a blocked set anymore, where $F_{x=v}$ is the formula we get by assigning a value $v \in \{True, False\}$ to a variable $x \in Vars(F)$ and simplifying the formula (remove satisfied clauses and remove unsatisfied literals from clauses).

Exercise 4: (Nonogram Challenge) [12(+12) Points] A Nonogram is a logic puzzle in which cells in a grid must be colored or left blank according to numbers at the side of the grid to reveal a hidden picture. Example (and the solution on the right):





The input is a single string looking like this (for the example above): `10,10:3,3;3,3;1,1,1;1,3,1;2,1,1,1;2,1,1,1;1,3,1;1,1,1,1;3,1,1,1;3,1,1,1;5,2;2,2,2;2,5;0;7;2,1;7;2;10;2;` The first two numbers are the width and heigth of the grid followed by the values separated by comma and semicolons. If the puzzle is unsatisfiable ouput `sol:UNSAT` otherwise print the solution in this format: `sol:1110000111;1110000111;...;1110101010` Implement a SAT solving based Nonogram solver. For a working solver you get 12 points. The fastest solver will receive a bonus of 12 points. In case of only 1 participant to get the bonus you must beat the media solution from previous years. Some example inputs here: `https://baldur.iti.kit.edu/sat/files/nonograms.txt`