Karlsruhe Institute of Technology

Practical SAT Solving

Lecture 9

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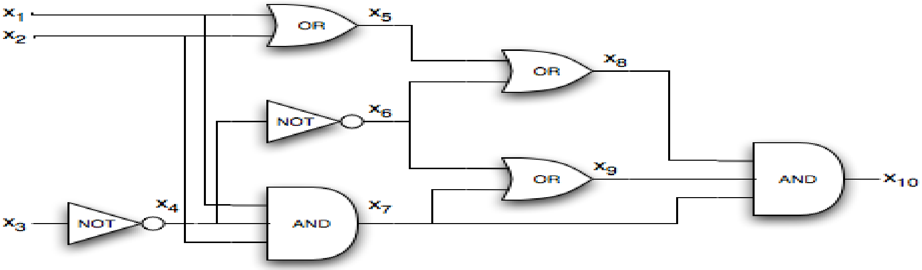
INSTITUTE FOR THEORETICAL COMPUTER SCIENCE



Lecture Outline

- VSIDS heuristic
- Clause forgetting
- Parallel SAT Solving

CDCL Algorithm

```
input  : Formula F in CNF
output: SAT / UNSAT

dl ← 0                          // initialize decision level
V ← ∅             // initialize trail (variable assignment)
while not all variables assigned do
    if unit_propagation(F, V) == CONFLICT then
        (c, bl) ← analyze conflict
        if bl < 0 then
            return UNSAT
        else
            add_clause(c)
            backtrack to bl
            dl ← bl
    else
        (x, b) ← pick branching literal
        dl ← dl + 1
        V ← V ∪ {(x, b)}
return SAT
```

Variable Selection in CDCL

- Previous heuristics (MOMS, Bohm's, etc.): global, "static"
  - E.g. MOMS: $S(x) = (f^*(x) + f^*(\overline{x})) \times 2^k + (f^*(x) \times f^*(\overline{x}))$
  - $f^*(x)$ is the number of occurrences of x in the smallest not yet satisfied clauses, k is a parameter
  - static: $S(x)$ often computed only at root node of search
  - global: based on whole CNF

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  - static: $S(x)$ often computed only at root node of search
  - global: based on whole CNF
- Idea for CDCL: Make heuristics more "focused"
  - try to find small unsatisfiable subsets
  - prefer variables that occurred in a recent conflict

VSIDS Heuristic

- VSIDS: Variable State Independent Decaying Sum
  - General approach: Compute score for each variable, select variable with highest score
  - Initial variable score is number of literal occurrences
  - New conflict clause c: Score is incremented for all variables in c
  - Periodically, divide all scores by a constant

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- First presented in SAT solver Chaff, 2001 [1]
- VSIDS (or a variant of it) implemented in most current CDCL solvers

VSIDS Example

Initial F:

$\{x_1, x_4\}$
$\{x_1, \overline{x_3}, \overline{x_8}\}$
$\{x_1, x_8, x_{12}\}$
$\{x_2, x_{11}\}$
$\{\overline{x_7}, \overline{x_3}, x_9\}$
$\{\overline{x_7}, x_8, \overline{x_9}\}$
$\{x_7, x_8, \overline{x_{10}}\}$

Scores:

$4 : x_8$
$3 : x_1, x_7$
$2 : x_3$
$1 : x_2, x_4, x_9, x_{10}, x_{11}, x_{12}$

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F with new learned clause added:

$\{x_1, x_4\}$
$\{x_1, \overline{x_3}, \overline{x_8}\}$
$\{x_1, x_8, x_{12}\}$
$\{x_2, x_{11}\}$
$\{\overline{x_7}, \overline{x_3}, x_9\}$
$\{\overline{x_7}, x_8, \overline{x_9}\}$
$\{x_7, x_8, \overline{x_{10}}\}$
$\{x_7, x_{10}, \overline{x_{12}}\}$ (new learned clause)

Scores:

$4 : x_8, x_7$
$3 : x_1$
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Implementation of VSIDS

- Possible: Keep list of variables sorted by score

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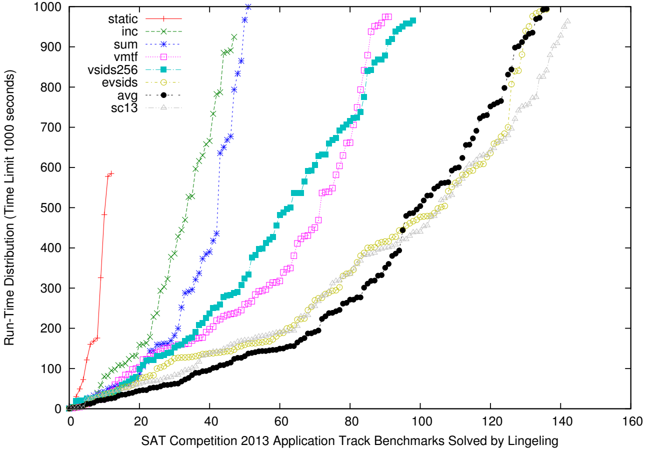
- Many implementations: Use priority queues
  - Operations: `insert_with_priority`, `pull_highest_priority_element`
- Often implemented as binary heaps
  - Insert: $\mathcal{O}(\log n)$
  - Delete: $\mathcal{O}(\log n)$
  - Peek: $\mathcal{O}(1)$

Variants of VSIDS

- Question: Why periodically divide scores?
- Answer: Give priority to recently learned clauses
- Chaff: half scores every 256 conflicts (“decay”); sort priority queue after each decay only
- Variants of VSIDS:
  - Berkmin’s strategy (Berkmin, 2002) – bump all literals in implication graph, divide scores by 4
  - VMTF: variable move to front (Siege, 2004)
  - CMTF: clause move to front (HaifaSAT, 2008)
  - eVSIDS – exponential VSIDS

Comparison of Heuristics



Learned Clause Removal

- Problem: Too many learned clauses!
  - ...and not all of them are helpful (e.g. subsumed clauses)
  - BCP gets slower, memory consumption
- Solution: Forget clauses after some time
  - also called Clause Database Reduction
  - size heuristics: discard long clauses
  - least recently used (LRU) heuristics: discard clauses not involved in recent conflict clause generation
  - "Glucose level": number of distinct decision levels in learned clauses (called LBD in original paper [2])

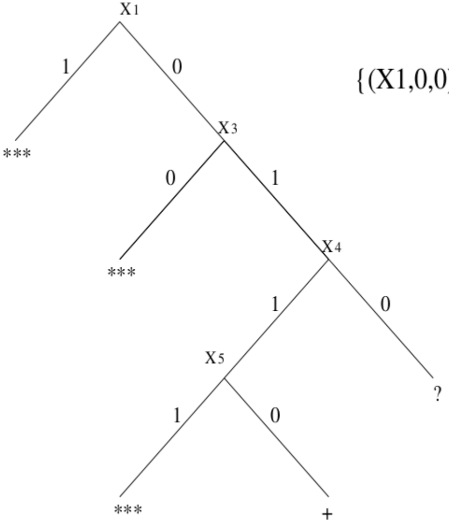
Partitioning – Böhm and Speckenmeyer

- 1994 First parallel implementation of DPLL
- completely distributed (no master and slave roles)
- A list of partial assignment is generated
- Each processors receives the entire formula and a few partial assignments
- Each Processors consists of
  - Worker (solve or split the formula, use the partial assignments)
  - Balancer (estimate workload, communicate, stopping)
- If a worker has nothing to do (all its partial assignments lead to UNSAT) a balancing process is launched.

PSATO – Zhang et al. 1994

- Centralized master-slave architecture
- Communication only between master and slaves
- Master assigns partial assignments based on the *Guiding Path*
  - Each node in the search tree is open or closed (closed means one branch is explored)
  - Master splits the open nodes and assigns job to slaves
- All processors can get stuck on unpromising branches

Guiding Path Example



guiding path

{(X1,0,0),(X3,1,0),(X4,1,1),(X5,0,0)}

*** : explored branch

+ : current node

? : remaining subtree

Satz – Jurkowiak et al. 2001

- The solver *Satz* improves PSATO the by adding *work stealing* for workload balancing
    - An idle slave request work from the master
    - The master splits the work of the most loaded slave
    - The idle slave and most loaded slave get the parts

2001 – Clause learning invented

Clause Sharing Parallel Solvers

- 2001, Blochinger et al.: PaSAT – the first parallel DPLL with "intelligent backtracking" and clause sharing
  - Similar to PSATO and SATZ: master slave, guiding path, randomized work stealing
- 2004, Feldman et al. – the first shared memory parallel solver
  - Multi-core processors started to be popular
  - uses same techniques as the previous solvers (guiding path etc.)
  - bad performance explained by high number of cache misses (DPLL/CDCL is otherwise highly optimized for cache)
- ... and many many more similar solvers

Cube and Conquer

Basic Idea

Generate a large amount of partial assignments (millions) and then assign each to one of the slaves.

- it is unlikely that any of the slaves will run out tasks
- The partial assignments are usually generated using a look-ahead solver (breadth-first search up to a limited depth)
- Examples of such solvers
  - march (Heule) + iLingeling (Biere) introduced the idea in 2011
  - Treengeling (Biere) – still state of the art for combinatorial problems
  - This kind of solver was used in the 200TB proof

Pure Portfolios

Basic Idea

Each processor works on the entire problem (no partial assignment restrictions). Each processors uses a (slightly) different solver (different heuristics, random seeds, etc.) All processors stop when one solver solves the problem.

- PPfolio – winner of Parallel Track in the 2011 SAT Competition
  - It is just a bash script that combines the best solvers from the 2010 Competition
  - The author: “it’s probably the laziest and most stupid solver ever written, which does not even parse the CNF and knows nothing about the clauses”
  - This kind of solvers is not allowed since then in SAT Competitions

Portfolios with Clause Learning

- Same as pure portfolio but clauses are shared
- Usually the same solver with different parameters is used for each processor
- 2009, Hamadi et al.[1]: ManySAT – the first solver using this idea (based on MiniSat)

[1] Microsoft® Research

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This is most successful approach since then

[1] Microsoft® Research

What Makes a Good Portfolio Solver

Two Pillars of Portfolios:

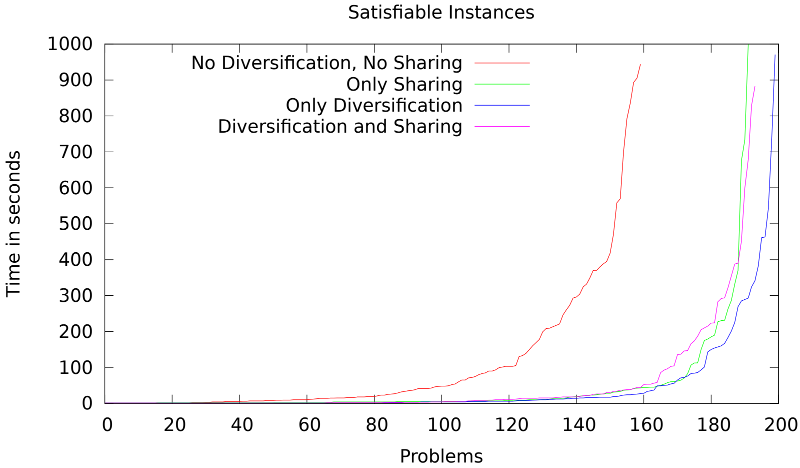
Diversification

- The search space of the solvers should not overlap too much
- Use different configuration values of heuristic parameters
- Partial assignment recommendations (no restrictions!)

Clause Sharing

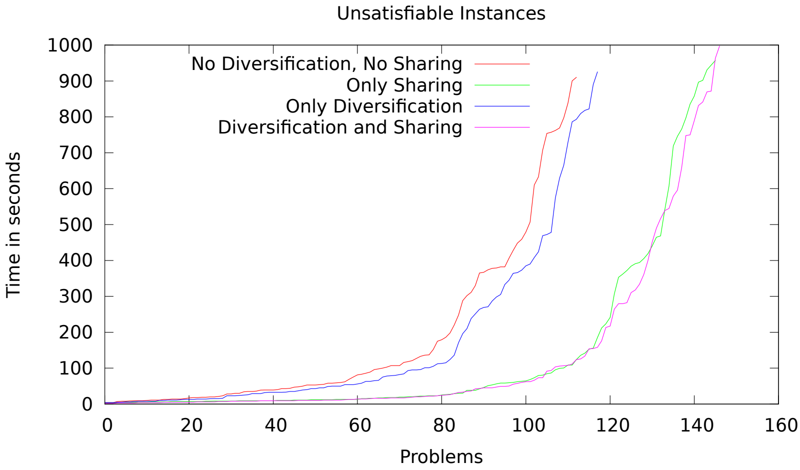
- Which clauses to share?
- How many?
- How often?
- How to implement efficiently?

Experiments – Random Satisf. 3-SAT

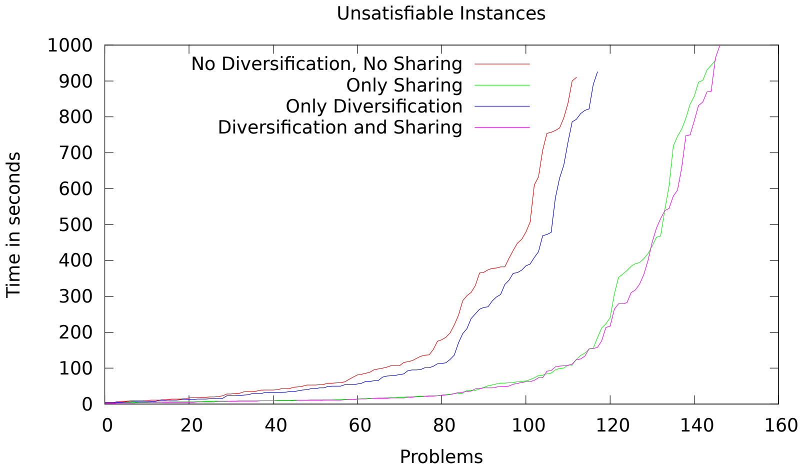


Advice for Satisfiable problems

Experiments – Random Unsat. 3-SAT



Experiments – Random Unsat. 3-SAT



- Clause sharing is important for UNSAT

A recent portfolio implementation

- HordeSAT – a Massively Parallel SAT Solver
- A scalable SAT solver for up to 2048 processors

HordeSAT Design Principles

- Modular Design
  - blackbox approach to SAT solvers
  - any solver implementing a simple interface can be used
- Decentralization
  - all nodes are equivalent, no central/master nodes
- Overlapping Search and Communication
  - search procedure (SAT solver) never waits for clause exchange
  - at the expense of losing some shared clauses
- Hierarchical Parallelization
  - running on clusters of multi-cpu nodes
  - shared memory inter-node clause sharing
  - message passing between nodes

Modular Design

Portfolio Solver Interface

```
void addClause(vector<int> clause);
SatResult solve(); // SAT, UNSAT, UNKNOWN
void setSolverInterrupt();
void unsetSolverInterrupt();
void setPhase(int var, bool phase);
void diversify(int rank, int size);
void addLearnedClause(vector<int> clause);
void setLearnedClauseCallback(LCCallback* clb);
void increaseClauseProduction();
```

- Lingeling implementation with just glue code
- MiniSat implementation, small modification for learned clause stuff

Diversification

Setting Phases – "void setPhase(int var, bool phase)"

- Random – each variable random phase on each node
- Sparse – each variable random phase on exactly one node
- Sparse Random – each variable random phase with prob. $\frac{1}{\#solvers}$

Native Diversification – "void diversify(int rank, int size)"

- Each solver implements in its own way
- Example: random seed, restart/decision heuristic
- For lingeling we used plingeling diversification

- Best is to use Sparse Random together with Native Diversification.

Clause Sharing

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Regular (every 1 second) collective all-to-all clause exchange

Exporting Clauses

- Duplicate clauses filtered using Bloom filters
- Clause stored in a fixed buffer, when full clauses are discarded, when underfilled solvers are asked to produce more clauses
- Shorter clauses are preferred
- Concurrent Access – clauses are discarded

Importing Clauses

- Filtering duplicate clauses (Bloom filter)
  - Bloom filters are regularly cleared – the same clauses can be imported after some time
  - Useful since solvers seem to "forget" important clauses

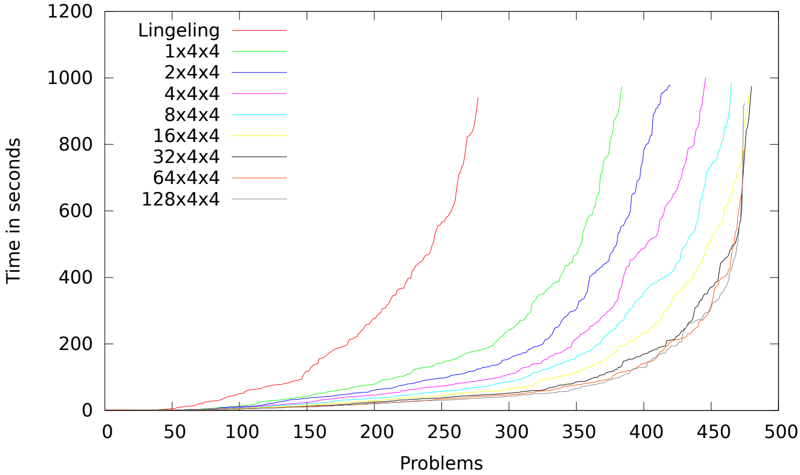
Overall Algorithm

The Same Code for Each Process

```
SolveFormula(F, rank, size)
  for i = 1 to numThreads do
    s[i] = new PortfolioSolver(Lingeling);
    s[i].addClauses(F);
    diversify(s[i], rank, size);
    new Thread(s[i].solve());

  forever do
    sleep(1) // 1 second
    if (anySolverFinished) break;
    exchangeLearnedClauses(s, rank, size);
```

Experiments – SAT 2011+2014



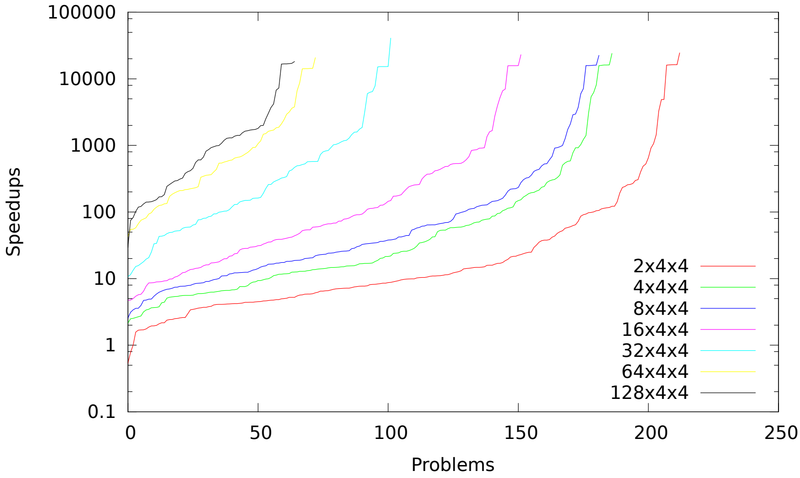
Experiments – Speedups

Big Instance = solved after $10 \cdot (\#\mathit{threads})$ seconds by Lingeling

Core Solvers	Parallel Solved	Both Solved	Speedup All Avg.	Speedup All Tot.	Speedup All Med.	Speedup Big Avg.	Speedup Big Tot.	Speedup Big Med.
1x4x4	385	363	303	25.01	3.08	524	26.83	4.92
2x4x4	421	392	310	30.38	4.35	609	33.71	9.55
4x4x4	447	405	323	41.30	5.78	766	49.68	16.92
8x4x4	466	420	317	50.48	7.81	801	60.38	32.55
16x4x4	480	425	330	65.27	9.42	1006	85.23	63.75
32x4x4	481	427	399	83.68	11.45	1763	167.13	162.22
64x4x4	476	421	377	104.01	13.78	2138	295.76	540.89
128x4x4	476	421	407	109.34	13.05	2607	352.16	867.00

Experiments – Speedups on Big Inst.

Big Instance = solved after $10 \cdot (\#threads)$ seconds by Lingeling



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