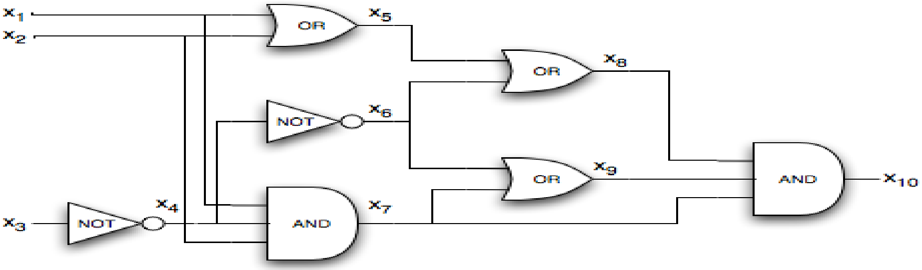
Practical SAT Solving

Lecture 7

Carsten Sinz, Tomáš Balyo | June 26, 2017

INSTITUTE FOR THEORETICAL COMPUTER SCIENCE



Lecture Outline

- Stålmarck's Method
- Advanced Techniques in DPLL
  - Restarts
  - Phase Saving

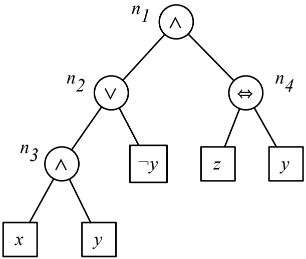
Stålmarck's Method [1]

- Input: Arbitrary formula F in propositional logic (need not be in CNF, $\Rightarrow$ and $\Leftrightarrow$ also allowed)
- Goal: Show unsatisfiability of F
- Preprocessing: Decompose formula tree into simple equations (triplets) T and a literal equivalence class R.
  ($R \subseteq L^0 \times L^0$ where $L^0 = L \cup \{0, 1\}$, R 'consistent')
- Basic processing steps: k-saturation ($k = 0, 1, \ldots$)
  0-saturation: simplification with triplet rules
  k-saturation ($k \geq 1$): case distinction, breadth-first search
- Developed by Gunnar Stålmarck ($\sim$1989), patented

Decomposition into Triplets

$F = ((x \wedge y) \vee \neg y) \wedge (z \Leftrightarrow y)$

Formula tree:



Initial equival. class: $\{n_1 = 0\}$
(to show unsatisfiability of F)

Triplets:

$$
\begin{aligned}
n_1 &= n_2 \wedge n_4 \\
n_2 &= n_3 \vee \neg y \\
n_3 &= x \wedge y \\
n_4 &= z \Leftrightarrow y
\end{aligned}
$$

Normalized triplets:
(only $\wedge$ and $\Leftrightarrow$)

$$
\begin{aligned}
n_1 &= n_2 \wedge n_4 \\
\neg n_2 &= \neg n_3 \wedge y \\
n_3 &= x \wedge y \\
n_4 &= z \Leftrightarrow y
\end{aligned}
$$

Stålmarck's Method: 0-Saturation

Given set of triplets T and literal equivalence class R apply derivation rules (deriving new literal equivalences):

$$\frac{p = q \wedge r \qquad p = 1}{\begin{matrix} r = 1 \\ q = 1 \end{matrix}} \; (A) \qquad \frac{p = q \wedge r \qquad p = \neg q}{\begin{matrix} p = 0 \\ r = 0 \end{matrix}} \; (D)$$

$$\frac{p = q \wedge r \qquad q = 0}{p = 0} \; (B) \qquad \frac{p = q \wedge r \qquad q = r}{p = q} \; (E)$$

$$\frac{p = q \wedge r \qquad q = 1}{p = r} \; (C) \qquad \frac{p = q \wedge r \qquad q = \neg r}{p = 0} \; (F)$$

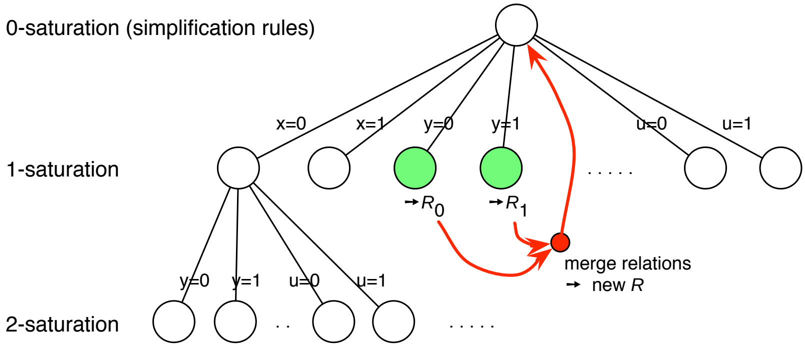
Stålmarck's Method: *k*-Saturation

Given formula F, represented as (T, R) (triplets and equiv. rel.) procedure `saturate` extends equivalence relation R:

```
EquivRel saturate(int k, TripletSet T, EquivRel R)
{
  if ( k = 0 ) return zero-saturate(T, R)
  forall x ∈ Var(T) not fixed in R do {
    R0 = saturate(k − 1, T, R ∪ {x = 0})
    R1 = saturate(k − 1, T, R ∪ {x = 1})
    R = R0 ∩ R1
  }
  return R
}
```

(`zero-saturate` returns all-relation if inconsistency was found)

k-Saturation: Graphical Illustration



Summary: Stålmarck's Algorithm

Input: Formula F represented as set of triplets T

(with n_1 representing top of formula tree)

Output: F satisfiable?

```
boolean stalmarck(TripletSet T)
{
  k = 0;  R = {n1 = 0}
  do {
    R = saturate(k, T, R)
    if ( R = all-relation ) return false
    else if ( R satisfies all triplets T ) return true
    else k = k + 1
  }
}
```

Restarts

- What is a restart?

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  - Clear the partial assignment
  - Unassign all the variables
  - Backtrack to level 0

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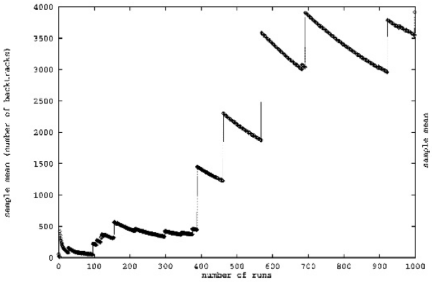
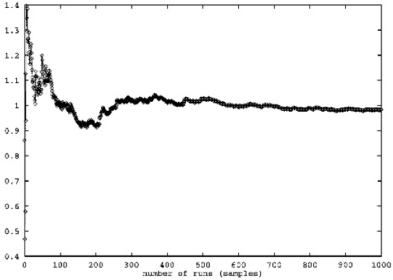
Restarts

- What is a restart?
  - Clear the partial assignment
  - Unassign all the variables
  - Backtrack to level 0
- Why would anybody want to do restarts in DPLL?
  - To recover from bad branching decisions
  - You solve more instances
  - Might decrease performance on easy instances

Restarts: Why?





Heavy-tail distribution

$$P[X > x] \sim C \cdot x^{-\alpha}$$

(for $0 < \alpha < 2$, $C > 0$)

Standard distribution

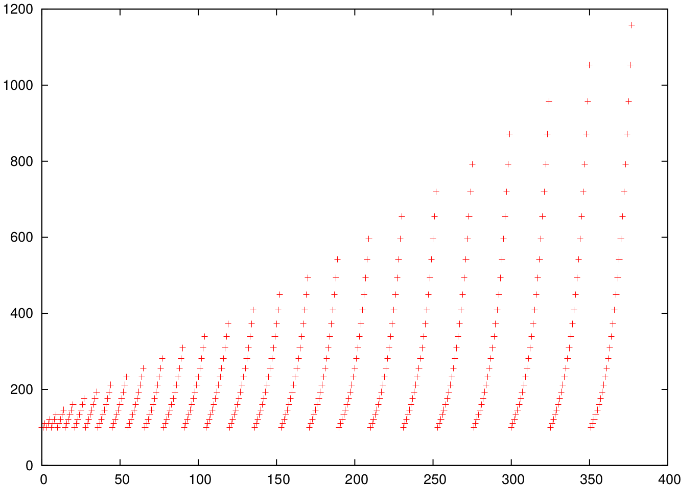
$$P[X > x] \sim \frac{1}{x\sqrt{2\pi}} e^{-x^2/2}$$

(Figures from Gomes *et al.*, 2000)

When to Restart?

- After a given number of decisions
- The number of decision between restarts should grow
  - To guarantee completeness
- How much increase?
  - Linear increase – too slow
  - Exponential increase – ok with small exponent
  - MiniSat: k-th restart happens after 100×1.1^k

Inner/Outer Restart Scheduling



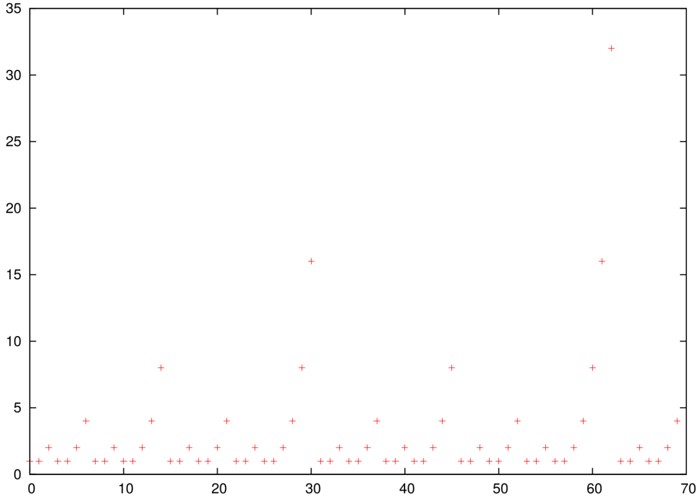
Inner/Outer Restart Scheduling

Inner/Outer Restart Algorithm

int inner = 100
int outer = 100

forever do
    . . . do DPLL for *inner* conflicts . . .
    restarts++
    if inner >= outer **then**
        outer *= 1.1
        inner = 100
    else
        inner *= 1.1

Luby Sequence Restart Scheduling



Luby Sequence

$$Luby = u \cdot (t_i)_{i \in \mathbb{N}}$$

$$t_i = \begin{cases} 2^{k-1} & \text{if } i = 2^k - 1 \\ t_{i-2^{k-1}+1} & \text{if } 2^{k-1} \leq i \leq 2^k - 1 \end{cases}$$

$$1, 1, 2, 1, 1, 2, 4, 1, 1, 2, 1, 1, 2, 4, 8, \ldots$$

Luby Sequence Restart Scheduling

Luby Sequence Algorithm

unsigned luby (**unsigned** i)
  for (**unsigned** k = 1; k < 32; k++)
    if (i == (1 $\ll$ k) - 1) **then return** 1 $\ll$ (k - 1)
  for (k = 1;; k++)
    if ((1 $\ll$ (k - 1)) <= i && i < (1 $\ll$ k) - 1) **then**
      return luby(i - (1 $\ll$ (k-1)) + 1);

limit = 512 * luby (++restarts);
... // run SAT core loop for *limit* conflicts

- Complicated, not trivial to compute

Reluctant Doubling

- A more efficient implementation of the Luby sequence
- Use the v_n of the following pair

$$(u_1, v_1) = (1, 1) \tag{1}$$

$$(u_{n+1}, v_{n+1}) = u_n \& - u_n = v_n ? (u_n + 1, 1) : (u_n, 2v_n) \tag{2}$$

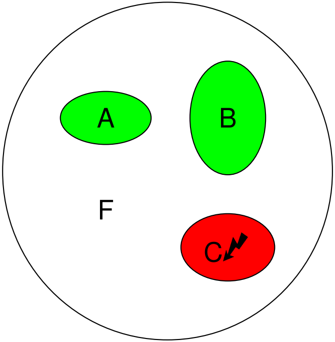
- Example: (1,1), (2,1), (2,2), (3,1), (4,1), (4,2), (4,4), (5,1), ...
- Invented by Donald Knuth

Phase Saving

First implemented in RSAT (2006)

`http://reasoning.cs.ucla.edu/rsat/`

- Observeration: Frequent *Restarts* decrease performance on some SAT instances
- Goal: Cache partial solutions to subsets of the formula and reuse them after a restart
- Idea: Remember last assignment of each variable and use it *first* in branching
- Result: *Phase Saving* stabilizes positive effect of restarts; best results in combination with *non-chronological backtracking* (later in this lecture)



Example: *A* and *B* are satisfied, search works on component *C*

References I

M. Sheeran, G. Stålmarck, A tutorial on Stålmarck's proof procedure for propositional logic, Formal Methods in System Design 16 (1) (2000) 23–58.
URL http://dx.doi.org/10.1023/A:1008725524946