Exercises for the Lecture Practical SAT Solving | Summer Semester 2016 | Assignment 1

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Exercise 1 (Graph Coloring, Empirical Evaluation) [10(+8) points]

In the lecture we presented a way to encode graph coloring problems as SAT problems using a direct encoding ("one-hot-encoding") for vertex colors.

Alternatively, we could have used a binary encoding for vertex colors.

1. **[3 points]** Describe an encoding for checking whether an undirected graph $G = (V, E)$ can be colored with at most k colors using a binary encoding for vertex colors. You may assume that $k = 2^i$ for some $i \in \mathbb{N}$.
2. **[7 points]** Write an encoder (in a programming language of your choice) that encodes graph coloring problems using a binary encoding. Input is a graph representation in DIMACS format and the number of colors (which may be restricted to powers of two).
3. **[optional, 8 points]** Compare the direct encoding and your binary encoding (a) analytically, giving the number of clauses and variables that are needed, and (b) experimentally, comparing the runtimes of a SAT solver of your choice on five graphs from `http://mat.gsia.cmu.edu/COLOR/instances.html` (use the *.col instances, the *.col.b are in binary format).

Exercise 2 (Sudoku) [5(+10) points] Write an encoder (in a programming language of your choice) for the generalized Sudoku puzzle. The generalized Sudoku puzzle of order n is an $n^2 \times n^2$ grid, consisting of n^2 sub-blocks of size $n \times n$, to be filled with numbers $1, \ldots, n^2$, such that

- in each row each number occurs exactly once,
- in each column each number occurs exactly once, and
- in each sub-block each number occurs exactly once.

The well-known Sudoku problem (see `https://en.wikipedia.org/wiki/Sudoku`) is the same as the generalized Sudoku puzzle of order 3. Benchmarks and input format description can be found here `http://www.cse.unsw.edu.au/~fpt09/comp/benchmarks.html`.

The best encoding (solving the most instances and fastest using Glucose 3.0) will a bonus of 10 points.

Exercise 3 (van der Waerden Numbers) [5 points]

How many clauses are in the SAT encoding presented in the lecture for checking whether $W(2,k) > n$?